Elif Baga. *Hesab Biliminde Kılavuz: eş-Şemsiyye fi'l-Hisâb (Nizâmeddin en-Nîsâbûrî), İnceleme-Çeviri-Eleştirmeli Metin* [*Handbook in the Science of Arithmetic: al-Shamsiyya fī al-ḥisāb* (*Niẓām al-Dīn al-Nīsābūrī*), *A Study, Translation, and Critical Edition*]. Istanbul: Türkiye Yazma Eserler Kurumu Başkanlığı Yayınları, 2020. 319 pages. ISBN: 9789751745743.

*Zehra Bilgin**

The literature on the tradition of mathematics in Islam gives importance to *ḥisāb* (arithmetic) books. The translations and analyses of these texts and critical editions have contributed to understanding the mathematical knowledge produced in Islam. Elif Baga has prepared one of the first examples of this subject in the Turkish literature. Her work contains a critical edition, Turkish translation, and mathematical analysis of Nizām al-Dīn Nīsābūrī's (d. 730/1329-30) *al-Shamsiyya fi al-ḥisāb*. This work is a revised and expanded edition of Baga's master's thesis entitled "Nizām al-Dīn Nīsābūrī and a Critical Edition, Translation, and Historical Analysis of his Mathematical Work *al-Shamsiyya fī al-ḥisāb*" that she prepared at Sakarya University in 2007.

The work consists of three chapters. Chapter 1 presents a detailed account of Nīsābūrī's life and works, especially *al-Shamsiyya fī al-ḥisāb*, and explains the method she adopted in her critical edition and translation. Chapter 2 provides a mathematical analysis of this work, while Chapter 3 lastly presents its translation and critical edition. The book ends with some facsimile pages of the original copies, followed by an Arabic-Turkish glossary of mathematical terms.

Baga aims to remove the ambiguities about the author's life as much as possible by combining the biographical information about Nizām al-Dīn Nīsābūrī found in various sources. Based on the copy dates of Nīsābūrī's works

* Assist. Prof., Fatih Sultan Mehmet Vakıf University, History of Science Department.
Correspondence: zbilgin@fsm.edu.tr.

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https://orcid.org/0000-0002-6147-3018

and the reigns of the rulers to whom they were dedicated, the date of his death is determined to have occurred in the first half of the 8th/14th century. In addition, referring to the names of the places in his full name, she states him to probably be of Qum origin but to have also spent his productive years in Nishapur. Nīsābūrī, who was an indirect student of Nasīr al-Dīn al-Ṭūsī (d. 672/1274) through his teacher Quṭb al-Dīn al-Shīrāzī (d. 710/1311), was educated and later studied in the fields of astronomy, mathematics, linguistics, and *tafsīr*. His works have been widely taught for many years in Turkistan and Anatolia. Considering the rulers to whom he dedicated his works, Baga points out that the peaceful environment of the Ilkhanate period when Nīsābūrī lived had provided a suitable environment for his studies.

Baga's comments on the intellectual development of Nīsābūrī's works being based on the order they were published is noteworthy. Nīsābūrī first wrote his works on astronomy, then his mathematical works, and finally his *tafsīr* as his *magnum opus*. According to Baga, when preparing for this famous work, the author first mastered astronomy with the aim of understanding the universe and then mathematics being situated between the physical and metaphysical sciences, finally writing his commentary once he had progressed enough in science.

After giving Nīsābūrī's biography in Chapter 1, Baga introduces the subject of her study as his *al-Shamsiyya fi al-ḥisāb*. Firstly, she explains how *Shamsiyya* in the title of the book is Nīsābūrī's dedication of this work to Shams al-Dīn ʿAbd al-Laṭīf, the son of Rashīd al-Dīn Faḍl Allāh (d. 718/1318) and one of the leading historians and statesmen of the time. Based on this information, Baga examines the time when Shams al-Dīn ʿAbd al-Laṭīf had served the state to determine the date of the work, and predicts it to have must been written before 707/1307.

Shamsiyya was a mid-sized textbook taught during the Ottoman period in wide use until ʿAlī al-Qūshjī's (d. 879/1474) famous work on arithmetic and calculation, *al-Risāla al-Muḥammadiyya,* became the leading textbook of the Ottomans. The book may have also been taught in the Samarqand Madrasa which had a strong tradition of astronomy and mathematics during the Timurid period. After this summary of the literature, Baga continues her previous discussion about the authorship of the work and confirms the book to truly belong to Nīsābūrī through the pieces of evidence she has obtained from historical sources.

After discussing the history of the work, she begins with the presentation of its contents. *Shamsiyya* consists of an introduction, two chapters, and an addendum.

After defining the science of calculation and its subject in the introduction, the author introduces positive integers and fractional numbers and describes how to write numbers using Indian numerals. The following two chapters explain the main subject of the work and methods for performing calculations. The first section of Chapter 1 is devoted to calculating positive integers and describes the operations of doubling a number and dividing a number in half as well as four primary operations with numbers. These operations are found in most of the classical calculation books. All these calculation methods are explained with the help of tables.

The second section is devoted to fractional numbers, first classifying fractional numbers as simple, compound, and integer fractions. This classification is standard in classical calculation books. Then this section goes on to describing the doubling and halving of fractions as well as the four arithmetic operations for them, similar to the first section.

Chapter 2 deals with topics on advanced calculations and consists of four sections. The first section describes how to calculate exponential and radical numbers and the methods for finding roots up to the fourth degree. The second section is about *abjad* and *sittīnī* (base sixty) calculations, which are both used in astronomy. The third section deals with *masāha* (applied geometry). The last section of this chapter explains algebra and operations using polynomial expressions and solutions for first- and second-order equations. After completing the book, Nīsābūrī presented it to Jamāl al-Dīn al-Tībī, after which he added an addendum based on al-Tībī's evaluations. The addendum explains the double false calculation used in solving equations and the *mīzān* verification method. When reviewing the contents, Nīsābūrī can be said to have examined the practices of Indian calculations in general and made these methods easy to understand with the help of examples and tables. Baga also comments how the pedagogical approach traced throughout the work explains why it had become a textbook widely read over the centuries.

After introducing the contents, the section goes on to explain how the critical text and translation of the work was prepared. In addition to many different copies of the *Shamsiyya*, two Arabic and Persian commentaries, one Persian translation, and a *takmila (complementary book)* are also found. Baga gives the bibliographic information and a brief description of the work itself, its commentaries, translation, and *takmila* and explains the motives for choosing the copies used in the critical edition. After the technical explanations about the critical edition and translation, Baga begins the mathematical analysis of the work.

As in almost all classical mathematical works, no mathematical notation apart from Indian numerals is used in the *Shamsiyya*. Mathematical definitions and operations are expressed purely with words. For this reason, Baga wisely judges that giving a mathematical analysis using modern mathematical symbols in addition to a word-by-word Turkish translation would be useful. Thanks to this method, the content of the text will be able to be diffused to modern minds unaccustomed to mathematical expressions written out in words. Starting from the definition of numbers, this section explains the calculation methods, algebraic operations, and calculations and formulas related to geometric objects using symbols. The order of the contents is the same as the original work. Several drawings of geometric figures allow readers to visualize the area and volume formulas given in the section on geometry. Baga enriched the main text of the *Shamsiyya* with extensive footnotes and contributed to the historical and mathematical context of the work. In this respect, the analysis section also serves as a small-scale commentary.

An analysis similar to the mathematical analysis of this work is essential for the history of mathematics studies to make classical mathematical works understandable for contemporary readers not only on a descriptive level but also in terms of content. Although various schools have different preferences on how to conduct this analysis, symbols are well known to be a convenient method. However, the technique that Baga follows in this work needs some improvements in terms of mathematical readability. The beginning of the analysis section states that, because the translation of the original work was written entirely in literal terms, only symbols are used in the mathematical analysis part. Although appropriate symbols are a requirement of modern mathematical literacy, Baga practically prefers to not use any literal expressions while writing out the mathematical expressions. Instead of making reading the text as easy as desired, this choice makes it difficult as deciphering a text consisting only of symbols is like reading a text written in a different alphabet. Verbal explanations and symbols would make the analysis text effective for readers of all levels and fields and allow it to be read without being tiresome.

The last part of the book provides the translation and critical edition of the *Shamsiyya* on facing pages. This method is a good choice for readers who want to follow the original text and translation together. The preferences for mathematical terms used in both the translation and mathematical analysis also needs to be mentioned. In the history of science studies, scientific words in classical texts should be transferred to a contemporary language accurately and in a way that fully corresponds to their meaning. One cannot expect all terms in a mathematical text from the classical period to be reciprocated precisely in today's mathematical language. Baga also draws attention to this issue, identifies equivalent expressions as much as possible, and explains with footnotes when necessary. She also explains some frequently used terms in a glossary at the end of the book. The contribution of this attempt to the classical mathematics literature in Islamic civilization is admirable. However, more research and analyses are necessary for some terms.

Two words commonly used in the text, *munṭaq* and *aṣamm* seem ambiguous in meaning. These two concepts are antonyms/complements of each other. Baga states that she uses the word *aṣamm* to mean prime number (a number that is not divisible by any number other than 1 and itself) when used in the context of division and to mean an irrational number (a number that cannot be written as a ratio of two integers) when involving radical operations. Accordingly, *munṭaq* numbers are numbers that are not *aṣamm* numbers. However, the use of *munṭaq* in the original text is not that simple. *Munṭaq* is first mentioned in the section on fractional numbers and is used for fractions whose numerator is 1 and whose denominator ranges between 2 and 10. These are called basic fractions, and other fractional expressions can be written as the sum of these fractions. Baga repeatedly uses the terms non-prime and rational for *munṭaq* in this section and sets the word prime number as an equivalent for *aṣamm* numbers. This approach results in some contradictions. Baga seems to use the contemporary definitions of these mathematical terms for the prime and rational numbers instead of the meaning Nisābūrī intended. For example, "rational fraction" as a phrase appeared and is used in the text; however, this is incorrect; as fraction, which is defined as the ratio of two integers, and the phrase "being rational" are considered completely equivalent concepts. *Aṣamm* in the text is given with a literal translation as "a number that is not counted by any number except 1" and corresponds to the modern term "prime number". This usage is correct; however, note that this expression only takes denominators of the fractions in question into account. Namely, the adjective of prime is used to qualify the number in the denominator, not the fraction itself. If a direct equivalent for prime is specified using *aṣamm*, ambiguity will occur because there are non-prime numbers between 2 to 10. Again, when dealing with radical expressions, the word "irrational" corresponds to *aṣamm*. Thus, fractions with numerators of 1 and denominators from 2 to 10 can be concluded to be *munṭaq* (basic fractions). Meanwhile, fractions with a prime number in the denominator for all other fractions are *aṣamm*/prime, while fractions with non-prime denominators are *munṭaq* (non-prime) for radical expressions. Also, numbers that are a complete square are called *munṭaq* (rational), while numbers that are not complete square are defined as *aṣamm* (irrational).

Baga's work here makes a remarkable contribution to the Turkish literature on the field of the history of mathematics in Islam both because of introducing the mathematical work from an influential scholar of the Islamic mathematical tradition to today's scientific community as well as for being a meticulous and comprehensive example of a critical edition and analysis of a book on classical mathematics. When considering *Shamsiyya* as a common and effective textbook during the period in which it had been written and beyond, the importance of this study is evident in understanding the scientific accumulation of Islamic civilization, especially the level of education and methods in mathematical sciences.