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Al-Jaghmīnī's Short Tract on the Volumes of the Planetary and Stellar Bodies: *Editio princeps* and Translation*

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Abstract: This article examines and provides an Arabic critical edition and English translation of the short tract attributed to Maḥmūd al-Jaghmīnī (fl. 600/1200) that deals with the volumes of the celestial bodies and that may have been intended as a supplement to *al-Mulakhkhaṣ*, his introduction to Ptolemaic theoretical astronomy. The work focuses on the sizes of the planetary bodies without addressing distances. The reader is provided with various lists such as which planetary bodies are above and below the Sun, the rounded volumes of bodies compared to the Earth, their sizes in descending order according to these volumes, and the body size of each measured in cubic parasangs (this being a mathematical calculation based on a derived parasang value for the Earth's volume and the stated relative volume for each body). No sources are mentioned in the witnesses; however, Jaghmīnī evidently chose modified Ptolemaic values, despite the availability of both the *Almagest* and *Planetary Hypotheses* in the 13th century. Whether Jaghmīnī considered intermediary sources to be authentic Ptolemaic values or not is unclear. Three of the four manuscript witnesses used for the edition also include a brief additional section on measurement, which is an excerpt from Sinān Pāshā's 15th-century gloss on Qāḍīzāde's commentary on *al-Mulakhkhaṣ*.

Keywords: Jaghmīnī, Islamic astronomy, distances and sizes of the celestial bodies, planetary volumes, Ptolemaic astronomy, *Planetary Hypotheses*, *Almagest,* measurement, Qāḍīzāde, Sinān Pāshā

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Introduction

Maḥmūd al-Jaghmīnī al-Khwārizmī (fl. 600 H/1200 CE) wrote multiple Arabic scientific works on astronomy, arithmetic, astrology, and medicine under the auspices of the Khwārizm-Shāhs (r. 470-628 H/1077-1231 CE) in Central Asia.[1] Two of his compositions became prominent textbooks that played an important role in scientific education and were disseminated widely throughout the Islamic world and South Asia for centuries: his medical treatise *al-Qānūnča* [The Little Canon], an abridgement of Ibn Sīnā's (d. 428 H/1037 CE) compendium *al-Qānūn fī al-ṭibb* [The Canon of Medicine]; and *al-Mulakhkhaṣ fī ʿilm al-hayʾa al-basīṭa* [Epitome of the Discipline of *hayʾa* Simplified], an introduction to Ptolemaic theoretical astronomy.[2] This article examines a short tract that was attributed to him, which deals with the volumes of the celestial bodies and may have been intended as a supplement to *al-Mulakhkhaṣ fī ʿilm al-hayʾa al-basīṭa* (herein simply referred to as *al-Mulakhkhaṣ*).

The question of authorship is made problematic because of a lack of internal attribution, an absence of references to the work in the premodern bio-bibliographical literature, and an attestation of authorship that was clearly appended to the text by someone other than the author. However, without being definitive, I would say the current evidence, given in what follows, argues for Jaghmīnī's authorship.

Jaghmīnī composed *al-Mulakhkhaṣ* in 602-3 H/1205-6 CE at the behest of Imām Badr al-Dīn al-Qalānisī,[3] who proposed that he compile a succinct work on *hayʾa*. As was understood in the Islamic astronomical tradition, a *hayʾa* treatise focuses on the external aspect of the bodies and offers a physical structure or configuration (*hayʾa*) of the universe, both for the celestial and the sublunary terrestrial regions.[4] While *al-Mulakhkhaṣ*, being a *hayʾa basīṭa* (simplified *hayʾa*) work, lacks geometrical proofs and mathematical derivations, Jaghmīnī also omitted any

1 For a list of Jaghmīnī's works, see Sally P. Ragep, *Jaghmīnī's Mulakhkhaṣ* (New York, 2016), 281–83 (Appendix I).

2 See S.P. Ragep, *Jaghmīnī's Mulakhkhaṣ*; and S.P. Ragep, "Jaghmīnī's *Qānūnča*," in *Transforming Medical Education*, eds. Delia Gavrus and Susan Lamb (Montreal, 2022), 54–85.

3 Badr al-Dīn Muḥammad ibn Bahrām ibn Muḥammad al-Qalānisī al-Samarqandī, who hailed from a prominent Damascene family (the Banū Qalānisī), was known as the author of a pharmaceutical treatise titled *Aqrābādhīn al-Qalānisī* (composed ca. 590 H/1194 CE) (see S.P. Ragep, *Jaghmīnī's Mulakhkhaṣ*, 16–19).

4 On *hayʾa* as a genre of astronomical writing, see F. Jamil Ragep, *Naṣīr al-Dīn al-Ṭūsī's Memoir on Astronomy (al-Tadhkira fī ʿilm al-hayʾa)*, 2 vols. (New York, 1993), 1:33–41. See 38–41 for an explanation of the external aspect of the bodies.

discussion of planetary sizes and distances of the celestial bodies that is often included in a chapter or section of a *hayʾa* treatise.[5]

According to the brief note found in all four manuscript witnesses, this is a work that Jaghmīnī wrote "at the time he completed the composition of *al-Mulakhkhaṣ*."[6] The implication seems to be that this short tract is related to *al-Mulakhkhaṣ*, perhaps as an appendage. Now one might claim that these remarks were added later and were based on a conjecture by a glossator whose note was then picked up in the four witnesses. However, contrary to this is the further comment that the dedicatee of either *al-Mulakhkhaṣ*, this tract, or perhaps both, was Badr al-Dīn al-Qalānisī.[7] Of the over 100 extant manuscript witnesses of *al-Mulakhkhaṣ*, the vast majority omit the dedication to Qalānisī. Indeed, the predominant text that circulated after the 13th century not only omits Qalānisī but also exhibits considerable tampering as far as parameters are concerned.[8] Furthermore, the most ubiquitous commentary on *al-Mulakhkhaṣ*, that of Qaḍīzāde al-Rūmī (d. ca. 835 H/1440 CE), also fails to mention Qalānisī.

What this argues for is that the writer of the note was familiar with the earliest version of *al-Mulakhkhaṣ* as well as, presumably, with the history of this short tract. One possible explanation for the sketchy nature of the tract (i.e., the lack of any conventional incipit or explicit) is that it was a rough draft meant to be appended to *al-Mulakhkhaṣ*. For whatever reason, Jaghmīnī never finalized it,[9] and *al-Mulakhkhaṣ* remained without a section on sizes and distances. This would explain the errors and incoherence, as well as the note that seeks to explain the status of

5 Some examples of *hayʾa* works that include sizes and distances are: ʿAbd al-Jabbār al-Kharaqī's *Muntahā* (composed ca. 526 H/1132 CE in Arabic), bk. II, ch. 17 (On Distances and Sizes); Naṣīr al-Dīn al-Ṭūsī's *Risālah-i Muʿīniyya* (composed 632 H/1235 CE in Persian), bk. IV (On Determining Distances and Sizes of Bodies, in Six Chapters); and Ṭūsī's *al-Tadhkira*, an Arabic reworking of the *Risālah-i Muʿīniyya* and its Supplement (final version completed 672 H/1274 CE), bk. IV (On Finding the Measurements of the Distances and the Bodies). See Hanif Ghalandari, ed., *Muntahá al-idrāk fī taqāsīm al-aflāk* (Tehran, 2020), 229–41; F.J. Ragep, *Ṭūsī's Memoir*, 1:310–41; and al-Ṭūsī, *al-Risāla al-Muʿīniyya (al-Risāla al-Mughniya) and its Supplement*, eds. Sajjad Nikfahm-Khubravan and Fateme Savadi, vol. 1, Critical Edition of the Persian Texts (Tehran, 2020), 175–91. https://ismi.mpiwg-berlin.mpg.de/page/muiniyya-edition-2020

6 For the text and translation of this passage in each manuscript witness, see the editions below.

7 The Arabic is ambiguous regarding which work is the subject of the dedication; see below for the text.

8 S.P. Ragep, *Jaghmīnī's Mulakhkhaṣ*, 2–4, 69–71.

9 One should recall that this had been a tumultuous period in the regions of Khurāsān and Khwārizm, and much of the literary tradition was lost or scattered haphazardly. See the seminal work of W. Barthold, *Turkestan Down to the Mongol Invasion*, 3rd ed. (London, 1968), esp. chs. 3 and 4. See also, S.P. Ragep, *Jaghmīnī's Mulakhkhaṣ*, 24–25.

the text. The evidence is not definitive, but there seems little reason to doubt the authenticity of the note and thus Jaghmīnī's authorship.

Another more technical reason to claim Jaghmīnī's authorship has to do with the mistakes found in the tract as discussed more fully below. The 13th-century works by Naṣīr al-Dīn al-Ṭūsī (d. 672 H/1274 CE), Quṭb al-Dīn al-Shīrāzī (d. 710 H/1311 CE), and others proficiently deal with sizes and distances, so it would be odd to find the types of errors in the work under discussion in astronomical texts after the 13th century. Finally, one should also remember that Jaghmīnī was a popularizer, not a first-rank Islamic astronomer, so it is not surprising that he could introduce a major error in his work. Mistakes of various sorts were not that uncommon in astronomical works, especially before the canonization of *hayʾa* in the 13th century; and as we shall see below, even an astronomer as preeminent as al-Bīrūnī could make major blunders.

Thus, for the above reasons, I believe Jaghmīnī's authorship can be conventionally accepted unless evidence to the contrary is forthcoming.

My original expectation was that the work would display Jaghmīnī's pedagogical ability to simplify the difficult subject of planetary distances and sizes concisely and accurately. However, on closer examination, it reads more like a compilation of notes rather than a coherent treatise.

As mentioned, all four manuscript copies used for the Arabic critical edition attribute authorship to Jaghmīnī; however, within the work itself, no internal title or statement of authorship is found to be present. After a brief invocation, the work focuses on the sizes of the planetary bodies. Distances are not treated, and consequently no clarification is found regarding how the numerical values were calculated. Furthermore, no sources are mentioned; rather, the reader is provided a *fait accompli* of various listings: the planetary bodies above and below the Sun; the rounded volumes of bodies compared to the Earth; their sizes in descending order according to these volumes; and the body size of each as measured in cubic parasangs, this being a mathematical calculation based on Jaghmīnī's derived cubic parasang value for the Earth's volume and his stated relative volume for each body. Three of the four manuscript witnesses also include an excerpt on measurement from the *Gloss* (*Ḥāshiya*) by Sinān Pāshā (d. 891 H/1486 CE) on Qāḍīzāde al-Rūmī's commentary on *al-Mulakhkhaṣ*.

Since Jaghmīnī is silent on his authorities, one can only speculate about possible sources based on the information as presented, the corpus of inherited material on the subject,[10] and the sources that can be identified from *al-Mulakhkhaṣ*, which, at least according to the note, was completed shortly before this tract. In *al-Mulakhkhaṣ*, Jaghmīnī specifically cites Ptolemy (fl. 140 CE) and his *Almagest*, al-Battānī (d. 317 H/929 CE), and the *zīj* literature.[11] Jaghmīnī also alludes to Ptolemy's *Geography*, a work written after the *Almagest*.[12] Finally, textual evidence indicates that Jaghmīnī depended on ʿAbd al-Jabbār al-Kharaqī's *al-Tabṣira fī ʿilm al-hayʾa*.[13]

The Volumes of the Bodies

Jaghmīnī's rounded numerical values for the volumes of the bodies using the Earth as the base unit clearly derive ultimately from Ptolemy's *Almagest* and/or *Planetary Hypotheses* (see Table 2). Because the *Almagest* deals only with the volumes of the Earth, Moon, and Sun and not with the other planetary bodies, Jaghmīnī not surprisingly would depend on Ptolemy's *Planetary Hypotheses* for the remaining values, a work cited under various titles such as *Kitāb al-iqtiṣāṣ* or *Kitāb al-manshūrāt*[14]

10 For an overview of summary accounts of astronomy before *al-Mulakhkhaṣ* (ancient and Islamic forebears), see S.P. Ragep, *Jaghmīnī's Mulakhkhaṣ*, 32–65. See also Guillaume Loizelet, who provides an in-depth study on the topic of planetary sizes and distances up to and including al-Bīrūnī (d. ca. 442 H/1050 CE). His discussions on the ancient and Islamic Ptolemaic traditions include an analysis of diverse pre-Islamic sources, their transmission, and the Arabic texts from Abū Maʿshar, al-Battānī, al-Bīrūnī, al-Farghānī, al-Qabīṣī, Thābit ibn Qurra, and al-Ṣaghānī ("Mesurer et ordonner les astres d'al-Farghānī à al-Bīrūnī: la tradition arabe du *Livre des Hypothèses* de Ptolémée [IXe-XIe s.]. Avec une édition et une traduction française du chapitre X.6 d'*al-Qānūn al-Masʿūdī* d'al-Bīrūnī," unpublished PhD thesis, University of Paris, Dec. 2021). For an extensive list of scholars who've written on the subject, see Mohammad Bagheri, Jan P. Hogendijk, and Michio Yano, "Kūshyār ibn Labbān Gīlānī's Treatise on the Distances and Sizes of the Celestial Bodies," *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften* 19 (2010-2011): 77–120, on 78–79; and J.P. Hogendijk, "Al-Ṣaghānī's Treatise on the Distances, Volumes and Surface Areas of the Planets and Fixed Stars," *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften* 20–21 (2012-2014): 1–29, on 2.

11 For references in *al-Mulakhkhaṣ* to Ptolemy, see II.1[2] (148–49) and II.3[9] (172–73); to al-Battānī, see II.3[9] (172–73); and to *zīj*es (astronomical handbooks), see I.2[10] (104–5) and II.3[7] (170–71). See also S.P. Ragep, *Jaghmīnī's Mulakhkhaṣ*, 253, 268, 278.

12 See S.P. Ragep, *Jaghmīnī's Mulakhkhaṣ*, II.1[2] (148–49) and 36–37, 268.

13 For example, in *al-Mulakhkhaṣ* (II.3[5]) (166–67), Jaghmīnī's exercise on using an astrolabe to determine the *qibla* bearing is strikingly similar to Kharaqī's passage in the *Tabṣira* (cf. Istanbul, Süleymaniye Lib., Laleli MS 2141, bāb 12, ff. 55a–56a). See also S.P. Ragep, *Jaghmīnī's Mulakhkhaṣ*, 63–64, 253, 277–78.

14 In his *Kitāb al-Fihrist* (composed 377 H /987 CE), the bibliographer Ibn al-Nadīm lists among Ptolemy's works *Kitāb iqtiṣāṣ aḥwāl al-kawākib* (vol. 2, ch. 7.2, p. 216; Engl. trans., 640). Al-Bīrūnī uses the title *Kitāb al-manshūrāt* in three of his treatises: *Kitāb al-Tafhīm, al-Qānūn al-Masʿūdī*, and *Kitāb fī taḥqīq mā lil-Hind*. See *Kitāb al-Tafhīm,* trans. R. Ramsay Wright (London, 1934), 115 [205] [Arabic facsimile], 151 [Persian text] (Tehran, 1983-84)]; *al-Qānūn al-Masʿūdī,* 3 vols (Hyderabad, 1954-56), 3:X.6 (1307, 1308); and *Kitāb fī taḥqīq mā lil-Hind* [his book on *India*], 2 vols. (London, 1910) 2:69 (ch. 55: On the Order of the Planets, their Distances and Sizes); [Arabic, p. 236]. Cf. Loizelet, "Mesurer et ordonner les astres," 277–78; and Willy Hartner, "Mediaeval Views on Cosmic Dimensions and Ptolemy's *Kitāb al-Manshūrāt*," in *Mélanges Alexandre Koyré,* 2 vols. (Paris, 1964), 1:254–82, esp. 257–58, 278–82.

and whose content Jaghmīnī was presumably aware of, either directly or due to various predecessors such as Abū Rayḥān al-Bīrūnī[15] and Ibn al-Haytham[16] who both had flourished some two centuries prior in the 10th/11th centuries.

However, the discrepancies indicate that Jaghmīnī had not simply copied Ptolemy's numbers; rather, the values from the *Planetary Hypotheses* most likely entered the Islamic world initially through a process of intermediation rather than directly, as Guillaume Loizelet has recently pointed out.[17] Also, despite the availability of both the *Almagest* and *Planetary Hypotheses*, Jaghmīnī, writing at the beginning of the 13th century CE, evidently chose modified values, possibly based on Abū Maʿshar (d. 272 H/886 CE) and Thābit ibn Qurra (d. 288 H/901 CE), the latter having been reported by al-Ṣaghānī (d. 379 H/990 CE). Whether Jaghmīnī thought these were authentic Ptolemaic values or not is unclear.

The two parts of the *Planetary Hypotheses*, originally composed in Greek, were available in an anonymous Arabic translation, supposedly corrected by Thābit ibn Qurra, and, if so, accessible as early as the 9th century CE.[18] However, no definitive evidence exists on whether Thābit had direct access to the *Planetary Hypotheses* or whether the work circulated in the 9th century.[19] Likewise, Thābit's 9th-century contemporary, al-Farghānī (d. 247 H/861 CE), had probably relied on an intermediary source for the values in his popular 30-chapter compendium on the science of the stars (*Jawāmiʿ ʿilm al-nujūm*). Farghānī devoted two chapters (21 and 22) to the subject of the distances and volumes of the planetary bodies and mentioned twice

15 In *al-Qānūn al-Masʿūdī* (2:634–35), Bīrūnī criticizes Ptolemy for some of his assumptions and ideas as going beyond the confines of the discipline of astronomy in the *Planetary Hypotheses*. Cf. F.J. Ragep, *Ṭūsī's Memoir*, 1:40.

16 Ibn al-Haytham refers to the *Planetary Hypotheses* as *Kitāb iqtiṣāṣ* in his *al-Shukūk ʿalā Baṭlamyūs*. See Ibn al-Haytham, *al-Shukūk ʿalā Baṭlamyūs* (Doubts about Ptolemy), edited by A.I. Sabra and N. Shehaby (Cairo, 1971; 2nd ed. 1996), 42 ff.; and A.I. Sabra, "An Eleventh-Century Refutation of Ptolemy's Planetary Theory," in *Science and History: Studies in Honor of Edward Rosen (Studia Copernicana XVI)* (Wrocław, 1978), 117–31.

17 Loizelet, "Mesurer et ordonner les astres," 7.7: 317–19.

18 Régis Morelon, "La version arabe du *Livre des Hypothèses* de Ptolémée," *MIDÉO* 21 (1993): 7–85, on 8–9. Goldstein ("The Arabic Version," 5) provides brief descriptions of the two extant Arabic manuscripts of the *Planetary Hypotheses*: London, British Museum, MS Arab, 426 [=British Library, Oriental MSS Add MS 7473], copied in 639/1242 from an exemplar copied in 531/1136: https://www.qdl.qa/en/archive/81055/vdc_100023677047.0x00000b; and Leiden, MS Arab 1155 (undated; the revision attribution to Thābit is on the cover page). For a historiographical introduction to the treatise, see Loizelet, "Mesurer et ordonner les astres," 4.1: 88–94.

19 This was Loizelet's determination after an extensive analysis ("Mesurer et ordonner les astres," 7.5: 300–9); the same conclusion was posited by Francis J. Carmody (*The Astronomical Works of Thabit b. Qurra* [Berkeley, 1960], 19).

that Ptolemy [in his *Almagest*] had dealt only with the bodies of the Moon and Sun.[20] Similar sentiments were echoed a century later by al-Ṣaghānī,[21] as well as by al-Qabīṣī in his treatise on the distances and sizes of the celestial bodies.[22]

For the Sun, Jaghmīnī used a value more in line with the *Planetary Hypotheses* rather than the *Almagest*, while the opposite is the case for the Moon. Again, this supports the hypothesis that he had used intermediary sources.

When comparing Jaghmīnī's planetary volumes with those in the *Planetary Hypotheses* for the other planets, several discrepancies may be noted: (1) Mercury's volume in the *Planetary Hypotheses* is 1/19,683 the size of the Earth as based on Ptolemy's stated diameter of 1/27 [$(1/27)^3$=1/19,683],[23] whereas Jaghmīnī's numerical value is 1/22,000. The difference is a computational divergence in rounding that may have been introduced by al-Farghānī.[24] The volume of 1/22,000 is a prevalent value, with Thābit ibn Qurra providing a notable exception in his *Simplified Almagest*, in which he gives the *Planetary Hypotheses*' value of 1/19,683;[25] (2) Jaghmīnī specifically refers to 15 large, fixed stars, whereas Ptolemy calls them the fixed stars of first magnitude without giving a specific number; (3) Jaghmīnī has the Moon larger than Venus, which reverses Ptolemy's listing of the order of these two bodies according to volume;[26] and (4) Jaghmīnī, unlike Ptolemy, includes in his

20 See al-Farghānī, *Jawāmiʿ ʿilm al-nujūm,* ed. Jacob Golius (repr. Frankfurt am Main, 1986), ch. 21: 80–82 and ch. 22: 83–85. Loizelet concludes that al-Farghānī was unaware of the *Planetary Hypotheses* and relied on an indirect source for the *Jawāmiʿ* ("Mesurer et ordonner les astres," 8.1: 323–24, on 324; 7.4: 291–300). Noel Swerdlow also determines that Farghānī was "ignorant" of the text ("Ptolemy's Theory of the Distances and Sizes of the Planets: A Study of the Scientific Foundations of Medieval Cosmology" [PhD diss., Yale University, 1968], 137–41, on 138, 140).

21 In the first chapter of his treatise, al-Ṣaghānī states: "In the Book [the] *Almagest*, Ptolemy only mentioned the distances and magnitudes of the two luminaries (sun and moon)" (Hogendijk, "Al-Ṣaghānī's Treatise," 3, 24 [Arabic], 5 [Engl. trans.]). It is ambiguous whether Ṣaghānī (or Farghānī) was aware of the *Planetary Hypotheses* based solely on pointing out the limitations of the *Almagest*.

22 See J. Hogendijk, "Al-Qabīṣī's Treatise on the Distances and Sizes of the Celestial Bodies: Edition and Translation," *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften* 20–21 (2012-2014): 169–233, on 170; 207 [Arabic], 177 [Engl. trans.].

23 Goldstein, "The Arabic Version," 33 (BM MS Arab 426, f. 91b [Arabic]), 8–9 [Engl. trans.]).

24 Swerdlow points out that a more precise diameter for Mercury, derived from the numbers used by Ptolemy, is $(1/27½)^3$. Thus 1/22,000 [$\approx(1/28)^3$] is the volume when rounded up to 1/28 instead of down to 1/27 (1/19,683). 1/28 is the value for the diameter of Mercury for Farghānī, and Swerdlow shows how it was derived ("Ptolemy's Theory," 3–5, 177).

25 Morelon, *Thābit ibn Qurra*, 14, line 4 (*L'Almageste simplifié*). However, Thābit gives 1/22,000 according to a report by al-Ṣaghānī (Hogendijk, "Al-Ṣaghānī's Treatise," 27 [Arabic], 8 [Engl. trans.]). Note that both Battānī and Bīrūnī also give non-Ptolemaic values for Mercury (see Table 2).

26 According to the *Planetary Hypotheses*, in which the volume of the Earth is 1, the volume of the Moon is 1/40, and the volume of Venus is 1/44, so the Moon should be larger; however, Ptolemy placed Venus

ordered list the smaller (sixth magnitude) fixed stars, placing them after Saturn and before Mars.[27]

As already mentioned, I know of two works containing volumes slightly modified from those in the *Planetary Hypotheses* that come strikingly close to Jaghmīnī's recorded values. However, no evidence exists to support that Jaghmīnī had been aware of either one; and, of course, other possible sources may also exist.

The first is an astronomical text on the bodies and distances that is attributed to Abū Maʿshar (fl. late 3rd/9th century) and was reproduced in the extant seventh volume of an encyclopedia by Ibn Rustah (fl. early 4th/10th century) titled *Kitāb al-Aʿlāq al-nafīsa* [Book of Precious Gems].[28] Two minor disparities are present between the values given by Jaghmīnī and Abū Maʿshar, both of which are explainable. The first is that Abū Maʿshar gave the volume of Mars as 1 15/49 the size of the Earth (not 1½), which is a computational error. Abū Maʿshar's true diameter compared to the Earth is identical to Ptolemy's numerical value in the *Planetary Hypotheses* (1⅐), so the volume should also be identical, with 1⅐ having been mistakenly squared instead of cubed. The second discrepancy relates to the volume given for the fixed stars of sixth magnitude. Abū Maʿshar gave 16, which was the value posited by Battānī, Kharaqī, and Kūshyār ibn Labbān. On the other hand, Jaghmīnī gave 15, which may simply have been the result of a computation based on his value of 95 for stars of first magnitude.[29]

ahead of the Moon (Goldstein, "The Arabic Version," 33 [BM MS Arab 426, f. 91b (Arabic), 9 (Engl. trans.)]). Ptolemy's error may be due to his ordering the bodies based on their true diameters, which for Venus is inconsistent with his stated volume of 1/44 (F.J. Ragep, *Ṭūsī's Memoir*, 2:528 [IV.7[4]]). For a volume of 1/44, the diameter should be ≈.28 (¼+1/30) rather than the given value of .3 (¼+1/20). Swerdlow gives ¼+1/30 in his Table 4.2 ("Ptolemy's Theory," cf. 171n2), although all Hebrew and Arabic manuscripts state ¼+1/20, according to Goldstein (12).

27 Ptolemy does give a Hipparchan value for the smallest star (1/30 the size of the Sun) but does not list it in his ranking according to volume (Goldstein, "The Arabic Version," [BM MS Arab 426, f. 90b (Arabic), 8 (Engl. trans.)]).

28 For Abū Maʿshar's text, see Ibn Rustah: Abû Alî Ahmed ibn Omar Ibn Rosteh, *Kitâb al-aʿlâk an-nafîsa VII*, ed. M.J. de Goeje, in *Bibliotheca geographorum Arabicorum* (Leiden, 1892), 17–22, esp. 20–22 (on the bodies). Just before Abū Maʿshar's text, Ibn Rustah (9–17) reproduced verbatim chapters 2-5 of al-Farghānī's *Jawāmiʿ*, making it plausible that he is doing the same for Abū Maʿshar's. Loizelet discusses Farghānī's compendium and Abū Maʿshar's work (mistakenly attributed to Ibn Rustah by Swerdlow) within an astrological context ("Mesurer et ordonner les astres," 6.2: 220–30; 8.3: 335–40). Cf. Swerdlow, "Ptolemy's Theory," 176–78.

29 A common method of computing the volumes of the fixed stars was based on a linearly decreasing magnitude starting with the first magnitude. For Jaghmīnī, this would have been dividing 95 by 6 (for the six magnitudes), and then subtracting the result of 16 continuously from 95 to obtain the volumes of the second-sixth magnitudes. So, the results for the first magnitude would be 95 and 15 for the

The second work containing the recorded volumes of the *Planetary Hypotheses* that correspond closely to Jaghmīnī's rounded values is a lost astronomical work attributed to Thābit ibn Qurra (d. 288 H/901 CE) partially preserved in a treatise by al-Ṣaghānī (fl. 4th/10th century). The single disparity is that the volumes for the fixed stars are omitted for both the fixed stars of the first and sixth magnitudes.[30]

In the *Simplified Almagest* (*Tashīl al-Majisṭī*),[31] Thābit intriguingly records either unmodified or rounded values from the *Planetary Hypotheses*, except for Venus, as a consequence of which Venus had a larger volume than the Moon and thus conformed with Ptolemy's stated listing of the descending order of the bodies.[32] If Jaghmīnī's objective was to faithfully transmit unaltered what he considered the authoritative recorded volumes for the planetary bodies attributed to Ptolemy,[33] the fact that he did not reproduce the numbers in Thābit's *Tashīl al-Majisṭī* may indicate that he had been unaware of the treatise or had not had access to it. Another possibility is that, for whatever reasons, he had trusted other sources more than the *Tashīl*.

A more speculative reason for Jaghmīnī discounting certain extant treatises may be that he had targeted works that relied on Ptolemaic values, especially re-

sixth. Two examples indicating the method's widespread use are by al-Farghānī (9th c.) in his *Jawāmiʿ*, ch. 22, and by Naṣīr al-Dīn al-Ṭūsī (13th c.) in his *Tadhkira*, bk. IV, ch. 7. See F.J. Ragep, *Ṭūsī's Memoir*, 1:340–41 (IV.7[3]) and 2:527–28; Swerdlow, "Ptolemy's Theory," 175; and Hogendijk, "Al-Ṣaghānī's Treatise," 14.

30 Hogendijk provides an edition, English translation, and commentary of the treatise, including a table (p. 12), comparing the planetary volumes in Thābit's lost astronomical work preserved by al-Ṣaghānī to Thābit's *Simplified Almagest*, to Ptolemy's *Planetary Hypotheses*, and to al-Farghānī's *Summary of Astronomy* (a.k.a. *Jawāmiʿ*) ("Al-Ṣaghānī's Treatise"). See also Loizelet, "Mesurer et ordonner les astres," 6.7: 260–66.

31 Morelon presents an analysis and critical Arabic edition with French translation of the work (*Thābit ibn Qurra*, XXXVIII-XLI, 1–17 [Traité 1: *L'Almageste simplifié*]). See also Loizelet, "Mesurer et ordonner les astres," 231–36 [6.3: *l'Almageste simplifié de Thābit ibn Qurra*].

32 Thābit's value of 1/37 for Venus versus Ptolemy's value of 1/44 is somewhat of a mystery. Loizelet ("Mesurer et ordonner les astres," 302) provides a chart comparing the values of the minimum and maximum distances (based on Earth radii) of the planetary bodies and the volumes contained in Thābit's *Simplified Almagest* with Ptolemy's *Planetary Hypotheses* (bk. I, pt. 2). The volume of Venus is the only outlier; the distances for Venus correspond (166 and 1079, respectively), but not the volumes. [See Morelon, *Thābit ibn Qurra,* 14, lines 2 and 10–13; cf. Goldstein, "The Arabic Version," 4, 11; 29 (BM MS 426, f. 89b [Arabic], 7 [Engl. trans.]).] Swerdlow ("Ptolemy's Theory," 176) points out that 1/37 is the value for Venus in Farghānī's *Jawāmiʿ* and suggests that Thābit may have "borrowed" it to correct Ptolemy's value of 1/44 to have Venus become larger than the Moon (1/40), thus conforming with Ptolemy's list of the descending ordering of the planets according to volume as stated in the *Planetary Hypotheses*. See also fn. 26 above.

33 Jaghmīnī may have been a member of a group of scholars that included the likes of Thābit ibn Qurra and Abū Maʿshar, who transmitted canonical values without critical analysis (a distinct tradition within Arab astronomy that is identified by Loizelet, "Mesurer et ordonner les astres," 323, 335, 424, 426).

garding measuring the Earth's size (a circumference of 24,000 miles based on 66⅔ miles per degree, as well as a diameter of approximately 7,636 miles).[34] This is what we find in Abū Maʿshar's text on bodies and distances,[35] but not in the works from Thābit,[36] Farghānī,[37] or Bīrūnī,[38] which use the Maʾmūnī values (a circumference of 20,400 miles based on 56⅔ miles per degree, as well as a diameter of approximately 6,500 miles).[39] As we will see, Jaghmīnī used the Ptolemaic values to derive his measurement for the Earth's body in parasangs, the Earth being the crucial unit for calculating the sizes of all the other planetary bodies.

In *al-Mulakhkhaṣ*, Jaghmīnī preferred the canonical parameters of Ptolemy despite the availability of treatises containing alternative values.[40] Thus, one might further speculate that Jaghmīnī had dismissed the volumes from al-Battānī and al-Kharaqī because of their deviation from Ptolemy, despite his dependence on them for other values in *al-Mulakhkhaṣ*. Since sizes and distances are absent in Kharaqī's *Tabṣira*, a work that Jaghmīnī used extensively for *al-Mulakhkhaṣ*, he would have needed to consult the extended treatment of the subject in Kharaqī's *Muntahā*, assuming that he wished to use Kharaqī as a source. Kharaqī had not given his own values for the planetary bodies but instead provided tables using the

34 Ptolemy states the measurement of the circumference of the Earth to be 180,000 stades in the *Geography* (VII.5) and 18 myriad stades in the *Planetary Hypotheses*. See J. Lennart Berggren and Alexander Jones, *Ptolemy's Geography: An Annotated Translation of the Theoretical Chapters* (Princeton, 2000), 110; and Goldstein, "The Arabic Version," 11, 31 (BM MS 426, f. 90b [Arabic], 7 [Engl. trans.]). A circumference of 24,000 miles was based on one mile being 7.5 stades, an equivalence that was introduced during the reign of the Ptolemies in Egypt.

35 Abū Maʿshar states that according to the Ancients, the value of the Earth's circumference is 24,000 miles (Ibn Rustah, ed. de G̲oeje, 17 [lines 22–23], 22 [lines 9–10]); the diameter is approx. 7,636 miles (18 [lines 3–4]); and a great circle on the Earth's surface is 66⅔ miles per degree (18 [lines 2–3]); 24,000 and 7,636 are repeated on 22 [lines 9–11].

36 Thābit accepts 56 miles per degree in al-Ṣaghānī's treatise (Hogendijk, 3, 10, 19). See also F.J. Ragep, *Ṭūsī's Memoir*, 2:507–8, 508n27. This is based on a diameter of 6,415 miles (Hogendijk, 7, 10, 25).

37 Farghānī cites al-Maʾmūn and gives his values for the Earth's surface (56⅔ miles per degree), circumference (20,400 miles), and diameter (approx. 6,500 miles) (ch. 8, *Jawāmiʿ*, 30–31).

38 Bīrūnī cites the Caliph al-Maʾmūn and gives his values for the Earth's surface (56⅔ miles per degree) and circumference (20,400 miles) (*Tafhīm*, 119 [208] [Arabic facsimile]; 160–64 [Persian text]). He also provides in parasangs the sizes of the Earth's circumference (6,800 parasangs) and diameter (2,163⅔ parasangs) [1 parasang = 3 miles] (*Tafhīm*, 118 [207] [Arabic facsimile], incorrectly given as 2163⅓ [correct value in Berlin, Staatsbibliothek, Petermann I MS 67, f. 34a]; 156, 156n1 [Persian text]). Note that for the fractional amounts, the Persian uses 4 *dānag* miles and 4 *dānag* parasangs (*dānag* meaning a sixth of anything).

39 For a nice summation of the Ptolemaic and Maʾmūnī measurements of the Earth, see Swerdlow, "Ptolemy's Theory," 213–15.

40 For example, see S.P. Ragep, *Jaghmīnī's Mulakhkhaṣ*, tables, 260–63.

values from Kūshyār ibn Labbān[41] and al-Bīrūnī,[42] both of which differ in varying degrees from what one finds in Ptolemy.[43] Likewise for Battānī, all his volumes differ from those in the *Planetary Hypotheses*.[44] On the other hand, given, as already mentioned, Jaghmīnī's own deviations from Ptolemy's values, it is apparent that he did not faithfully copy them from the *Planetary Hypotheses*; indeed, it cannot be established whether he had a copy of the *Planetary Hypotheses* or even knew what was in it. That Jaghmīnī's values are closest to Abū Maʿshar (via Ibn Rustah) and Thābit (via Ṣaghānī) would argue for his dependence on a simple, straightforward source without any extensive mathematical discussion involving distances.

Finally, the problematic nature of sizes and distances in many of these treatises is worth mentioning. If Jaghmīnī had consulted Bīrūnī (either directly or as transmitted by Kharaqī), he might have noticed that Bīrūnī had introduced a serious mistake regarding the size for the Earth's volume in cubic parasangs, an error Kharaqī had also reproduced in his *Muntahā*.[45] Bīrūnī and Kharaqī were not infallible,

41 Kharaqī provides values for distances and sizes of all the planetary bodies in a table titled Kūshyār's "corrected" ones (presumably emended from the Ptolemaic ones). See Ghalandari, *Muntahá*, 240–41; cf. Berlin, Staatsbibliothek, Landberg MS 33, f. 50b. For Kūshyār's values in his own works, see *al-Zīj al-Jāmiʿ* (bk. III, ch. 22), Leiden, Univ. Lib. MS Or. 8, f. 94a–b; and *Risāla fī al-abʿād wa-al-ajrām*, Kūshyār's treatise on the distances and sizes of the celestial bodies intended as a commentary to bk. III, ch. 22 of his *Zīj* (Bagheri, Hogendijk, and Yano, "Kūshyār," 111–18 [Arabic], 85–90 [Engl. trans.]). See also Loizelet, "Mesurer et ordonner les astres," 270.

42 Kharaqī also provided a table of "corrected" values from Abū Rayḥān's *Kitāb al-Tafhīm* of the nearest distances, diameters, and volumes of the planetary bodies (all in terms of the Earth), as well as their absolute volume measurements in cubic parasangs. See Ghalandari, *Muntahá*, 236–39; cf. Berlin, Staatsbibliothek, Landberg MS 33, f. 50a. For Bīrūnī's values in the *Tafhīm*, see 116–17 [206] [Arabic facsimile]; 154–55, and 158–59 [Persian]. Note that some minor differences occur between the Persian and Arabic versions. See also Swerdlow, "Ptolemy's Theory," 182–86, 187, esp. tables 4.7 and 4.8.

43 For the values of Bīrūnī, and Kūshyār, see Table 2 below.

44 For indications that Battānī had no knowledge of the *Planetary Hypotheses*, see Swerdlow, "Ptolemy's Theory," 143–46, 179–81; and Loizelet, "Mesurer et ordonner les astres," 6.5: 246–54; 8.2: 326–27. Note that Battānī mistakenly lists the Moon as larger than Venus, even though his relative volumes (Venus: ≈1/36; Moon: ≈1/39¼) would indicate otherwise (Nallino, *Al-Battānī*, ch. 50, 3:185 [Arabic], 1:123 [Latin]). For his values, see Table 2.

45 For the Earth's volume, Bīrūnī gives the exceedingly inaccurate value of 166,744,242 14/33 cubic parasangs (*Tafhīm*, 117 [206] [Arabic facsimile, Engl. trans.]; 158 [Persian text]) and elsewhere in the same work as 166,744,242 2/5 cubic parasangs (118 [207] [Arabic facsimile, Engl. trans., the latter mistakenly giving ⅕]; 157 [Persian text]). Kharaqī perpetuates the error (i.e., 166,744,242 14/33) in his table of Bīrūnī's value for the volume of the Earth (Ghalandari, *Muntahá*, 236; cf. Berlin, Staatsbibliothek, Landberg MS 33, f. 50a). In another context, Bīrūnī gives a more correct value of 5,305,498,589 4/5 cubic parasangs (*Tafhīm,* 119–20 [209] [Arabic facsimile, Engl. trans.]; 165 [Persian text]). For a reconstruction of Bīrūnī's computational error, see Swerdlow, "Ptolemy's Theory," 186, 187, 216–17. Swerdlow's masterful analysis provides the justification for reading the 14 33 in the manuscripts as a fraction (i.e., 14/33) rather than sexagesimally (i.e., 0;14,33). Note that 14/33 occurs in a table that Swerdlow seems to have missed; the number with 2/5, which Swerdlow thinks is Bīrūnī's only value, occurs in the text on the following page of the manuscript facsimile and is clearly just a rounded value for 14/33.

and neither was Jaghmīnī, who also made a serious blunder regarding the value for the Earth's volume (about which, see below). Apparently, errors in determining sizes for the planetary bodies (whether relying on either the Ptolemaic or Maʾmūnī values) were not uncommon.[46]

The Sizes of the Bodies

Jaghmīnī's list of sizes for the planetary and stellar bodies using the Earth's volume as one and the ensuing statement regarding their descending order according to these volumes are based on modified Ptolemaic values and presumably were gleaned from an extant treatise that had recorded them. However, Jaghmīnī's attempt to then detail the sizes of the bodies in cubic parasangs in accordance with Ptolemaic measurements was doomed to fail from the outset, because Jaghmīnī's value of 20,363,630⅓ parasangs for the Earth's volume (cubic parasangs) as the basis on which he calculated the sizes of all the other bodies is actually the Earth's surface area (square parasangs). One can only surmise that what contributed to Jaghmīnī's crucial error had been the rarity of treatises on determining the actual measurements of the surface areas and volumes of all the celestial bodies, especially ones providing sizes converted to parasangs.[47] Based on Jaghmīnī's reliance on Kharaqī's *Tabṣira* in *al-Mulakhkhaṣ*, one might expect that he had consulted Kharaqī's values for sizes in his *Muntahā*, which, as mentioned, were missing in the *Tabṣira*. Kharaqī had calculated 183,264,000 sq. miles[48] (which converts to 20,362,666⅔ sq. parasangs) for the Earth's surface area, which was based on Ptolemy's diameter and circumference; as was already mentioned, Kharaqī had also included a chart reproducing Bīrūnī's values that included the incorrect volumes of the celestial bodies in cubic parasangs. Despite the expectation that Kharaqī might have been a source, Jaghmīnī's 20,363,630⅓ (however interpreted) is only approximately Kharaqī's value, and Jaghmīnī's parasang volumes bear no relationship to Bīrūnī's.

46 Hogendijk, "Al-Ṣaghānī's Treatise," esp. 17n9.

47 Before Jaghmīnī, Bīrūnī seems to be the only writer to have attempted to give the volumes in terms of cubic parasangs (*Tafhīm,* 117 [206] [Arabic facsimile, Engl. trans.]; 158–59 [Persian text]). But due to a serious miscalculation in the Earth's volume (see fn. 45), all his values are considerably off; in any event, they have no relationship to Jaghmīnī's numbers. Abū Jaʿfar al-Khāzin (d. ca. 360/970) and al-Qabīṣī gave surface areas in square miles, but their numbers also bear no relationship to Jaghmīnī's. For the former, see Hogendijk, "Al-Ṣaghānī's Treatise," 9–10, 14–18, 28–29; for the latter, see Hogendijk, "Al-Qabīṣī's Treatise," 207, 230 [Arabic], 177, 203 [Engl. trans.].

48 For Kharaqī's citation of the Ptolemaic value of 183,264,000 sq. miles for the Earth's surface, see Ghalandari, *Muntahá*, 231 [417].

Jaghmīnī was not unique in positing a value without explaining its origin or how it had been derived.[49] Still, given the lack of transparency, Table 1 is my reconstruction of how the numerical value of 20,363,630⅓ parasangs could have been obtained for the Earth's surface area by relying on Ptolemaic measurement. That Jaghmīnī had assumed this value to be the Earth's volume remains a mystery; but he likely had obtained it from some unknown source as deriving it himself presumably would have made him cognizant of the fact that he was calculating the surface area.

Table 1.

Earth's Surface Area According to Ptolemy and Jaghmīnī

Ptolemy	
circumference	24,000 miles
diameter (24,000*7/22)	7,636 $^4/_{11}$ ≈7,636 miles
surface area [=c*d] (standard value)	183,264,000 [24,000*7,636] sq. miles
surface area (1sq. parasang = 9 sq. miles)	20,362,666 ⅔ sq. parasangs
Jaghmīnī	
surface area [=c*d] (using precise diameter)	183,272,727 $^3/_{11}$ [24,000*7636 $^4/_{11}$] sq. miles
surface area (modern calculation)	20,363,636 $^4/_{11}$ sq. parasangs[50]
surface area (Jaghmīnī's value given in manuscripts)	20,363,630 ⅓ sq. parasangs[51]

Jaghmīnī compounded his mistake by using his surface area value of 20,363,630⅓ parasangs for the Earth's volume to subsequently derive the numerical values for the volumes of the other celestial bodies. Despite being incorrect, many of the results are consistent mathematically, which seemingly confirms these

49 Recall that since Kūshyār ibn Labbān only provided results without computation in *al-Zīj al-Jāmiʿ* (bk. III, ch. 22), he composed *Risāla fī al-abʿād wa-al-ajrām* (Bagheri, Hogendijk, and Yano, "Kūshyār," 78).

50 20,363,636 $^4/_{11}$ sq. parasangs for the Earth's surface area is found in Ghiyāth al-Dīn Jamshīd al-Kāshī's *Sullam al-samāʾ* (pt. 1), composed 809 H/1407 CE (Istanbul, Süleymaniye Lib., Esad Effendi MS 2034, f. 16b) and ʿAbd al-ʿAlī al-Bīrjandī's *Sharḥ al-Tadhkira* (bk. IV, ch. 1), composed 913 H/1507 CE (Samsun, Gazi İl Halk Library, MS 810, f. 407b).

51 Exactly how Jaghmīnī arrived at 20,363,630⅓ or whether this is a copyist error is unclear; assuming that the $^4/_{11}$ has been rounded to ⅓, there is only a one-digit discrepancy with the modern, recalculated value (20,363,636 $^4/_{11}$), which is also found in Kāshī and Bīrjandī. See Table 3 for a comparison of the calculated values of the celestial bodies using the parameters 20,363,630⅓, and 20,363,636⅓; unfortunately, which parameter Jaghmīnī was using cannot be definitively established.

to be Jaghmīnī's intended values rather than a repeated copyist error. However, not all the values are computationally correct, and whether these are due to miscalculations or scribal miscopying is not at all clear. In any case, a computation for each volume is provided in Table 3 as well as in the translation footnotes.

On Measurement

Three of the four manuscript witnesses add a brief section on measurement from Sinān Pāshā's 15th-century *Gloss* (*Ḥāshiya*) on Qāḍīzāde al-Rūmī's commentary on *al-Mulakhkhaṣ* (whose authorship is noted in MS K, f. 132a). Jaghmīnī's use of the standard equivalence of 3 miles = 1 parasang is not in question; however, Sinān Pāshā's gloss on the parasang indicates an awareness of the ongoing disagreement regarding what constitutes a mile based on different values for a mile that existed in both pre-Islamic and Islamic times.[52] Sinān Pāshā points out an equivalence between the Ancient and Modern values, with the differences having to do with differing values for cubits and digits that cancel each other out. The Ptolemaic values are 1 mile = 3,000 cubits and 1 cubit = 32 digits, whereas the Maʾmūnī values are 1 mile = 4,000 cubits and 1 cubit = 24 digits. Thus, one mile was equal to 96,000 digits, whether Ancient or Modern.

Kharaqī also makes the Modern (i.e., Maʾmūnī) mile to be 96,000 digits, but according to him the Ancient (i.e., Ptolemaic mile) has 108,000 digits.[53] He arrives at the latter by making the Ancient mile equal to 3,000 cubits, each cubit being 36 digits. It is worth noting that Kūshyār ibn Labbān also claims the Ptolemaic value for a cubit to be 36 digits, not 32.[54] On the other hand, Naṣīr al-Dīn al-Ṭūsī in *al-Risāla al-Muʿīniyya* has Jaghmīnī's equivalence of Ancient and Modern miles, thus also departing from Kharaqī and Kūshyār.[55] A more thorough investigation of these conflicting values for the premodern mile and their implications is certainly warranted.

52 For details as they relate to the size of the Earth, see F.J. Ragep, *Ṭūsī's Memoir*, 2:501–10.

53 For the Maʾmūnī values (1 mile = 4,000 cubits; 1 cubit = 24 digits), see Ghalandari, *Muhtahá,* 230 [414]; for the Ptolemaic values (1 mile = 3,000 cubits; 1 cubit = 36 digits), see Ghalandari, *Muhtahá,* 231 [416]; cf. Berlin, Staatsbibliothek, Landsberg MS 33, f. 49b.

54 See Kūshyār's *Zīj al-Jāmiʿ,* bk. III, ch. 22 (Leiden, Univ. Library MS Or 8, f. 99b) and his *Risāla* (Bagheri, Hogendijk, and Yano, "Kūshyār," 107 [Arabic], and 82 [Engl. trans.]).

55 Al-Ṭūsī, *al-Risāla al-Muʿīniyya*. See 176–77, IV.1[2–3].

Table 2.

Sizes (in Earth Volumes)

BODY[56]	VOLUMES (with Earth as unit)									
	Jaghmīnī *On sizes of bodies*	**Ptolemy** *Planetary Hypotheses*[57] [*Almagest*[58]]	**Farghānī** *Jawāmiʿ*[59]	**Abū Maʿshar** [recorded text][60]	**Thābit** *Simplified Almagest*[61]	**Thābit** [Ṣaghānī report][62]	**Battānī** *Zīj*[63]	**Qabīṣī** *On sizes and distances*[64]	**Bīrūnī** *Tahfīm*[65]	**Kūshyār** *Zīj;Risāla*[66]
Sun	167	166⅓ [170] [67]	166	166⅜ [68]	166	166¼+⅛	166¼+⅛	170	167⅓ [69] [166⅓]	166¼+⅛

56 The listing of the bodies is in accordance with Jaghmīnī's ranking in descending volume size with the Earth's volume as 1.

57 See Goldstein, "The Arabic Version of Ptolemy's Planetary Hypotheses," 33 (BM MS Arab 426, f. 91b [Arabic text]); 9 [Engl. trans.].

58 See Toomer, *Ptolemy's Almagest,* 257, V.16 {*On the sizes of sun, moon* and *earth*}.

59 See *Jawāmiʿ ʿilm al-nujūm*, ch. 22, 83–85 [Arabic and Latin trans.].

60 The values are contained within a reproduced text on the celestial bodies and distances attributed to Abū Maʿshar by Ibn Rustah in his *Kitāb al-aʿlāq al-nafīsa* (see *Bibliotheca geographorum Arabicorum*, 7:20–22).

61 For Thābit's values in his *Tashīl al-Majisṭī* ("L'Almageste simplifié"), see R. Morelon, *Thābit b. Qurra*, 13–14 (critical Arabic ed. and French trans.).

62 Thābit's values are reported in a treatise by al-Ṣaghānī (see Hogendijk, "Al-Ṣaghānī's Treatise," 12 [table]; 27 [Arabic], 8 [Engl. trans.]; cf. Damascus, Ẓāhiriyya MS 4871, ff. 78b–79b).

63 See Nallino, *Al-Battānī*, ch. 30 (3:90–91 [Arabic], 1:60 [Latin trans.] for the Sun and Moon); ch. 50 (3:181–86 [Arabic], 1:120–24 [Latin trans.] for the remaining bodies).

64 See Hogendijk, "Al-Qabīṣī's Treatise," 174 [table]; 219, 222, 224, 225, 227, 229 [Arabic], 191, 194, 196, 198, 199, 201, 202 [Engl. trans.].

65 For Bīrūnī's values in *Kitāb al-Tafhīm*, see 116 [206] [Arabic facsimile] and 154–55 [Persian]; cf. Berlin, Staatsbibliothek, Petermann I MS 67, f. 33a. Kharaqī in his *Muntahā* also provides a table of Bīrūnī's sizes and distances; see Ghalandari, *Muntahá*, 236–39; cf. Berlin, Staatsbibliothek, Landberg MS 33, f. 50a. Bracketed values are the Persian variants; variants from the *Muntahā* are given in the footnotes.

66 For Kūshyār's values, see *al-Zīj al-Jāmiʿ* (bk. III, ch. 22), Leiden, Univ. Lib. MS Or. 8, f. 94a–b; for the *Risāla*, see Bagheri, Hogendijk, and Yano, "Kūshyār," 111–18 [Arabic], 85–90 [Engl. trans.]. Kharaqī in the *Muntahā* also provides a table of Kūshyār's sizes and distances; see Ghalandari, *Muntahá*, 240–41; cf. Berlin, Staatsbibliothek, Landberg MS 33, f. 50b. The variants from the *Muntahā* are given in the footnotes.

67 Ptolemy stated (*Almagest*, V.16) "the sun's volume is about 170 times that of the earth." The same number is in Proclus (5th c.), *Hypotyposis,* IV (Manitius, 132 [Greek]; 133 [German trans.]).

68 166+¼+⅛ or 166⅜ for the Sun's volume can be derived from Ptolemy's *Almagest*, V.16 by cubing Ptolemy's value of 5½ for the Sun's true diameter [$(5½)^3$ = 166.375].

69 Kharaqī has 167⅓ (Ghalandari, *Muntahá*, 237).

15 largest fixed stars	95	94+⅙+⅛	107	≈94½	94	-	105	91⅛	106;8′ [70]	94⅕
Jupiter	82	82½+¼+1/20	95	≈81½+¼	82	81½+¼	≈81	82¼ +⅔ (⅒)	95;14′	84¼+⅛ [71]
Saturn	80	79½	91	≈79½	79	79½	≈79	94⅙ [72]	92;8′	81⅕+⅙
smallest [fixed stars][73]	15	-	18	16	-	-	16	-	18;16′ [74]	16
Mars	[1½] [75]	1½	1½+⅛	≈1 15/49 [1½] [76]	1½	1½	‹1⅓	≈1⅔+¼	1;27′ [77]	≈1½

70 106;03 in the English translation is a misreading. Ghalandari, *Muntahá*, 239 has 180;3 (قف جـ), but this is most likely a copyist misreading of قو ح (i.e., 106;8).

71 Ghalandari has 84+⅕+⅙ for the volume of Jupiter (*Muntahá*, 241); however, 84+¼+⅛ is a variant in several manuscript copies. The fractional value ⅕+⅙ may be an error introduced by repeating the fractional amount of Saturn's volume of 81+⅕+⅙. In Landberg MS 33, f. 50b, a copyist has indicated that ¼+⅛ is a variant in another manuscript copy (خ) for ⅕+⅙.

72 Qabīṣī alone gives a volume for Saturn (94⅙ based on a diameter of 4½+½[⅒]) that is larger than Jupiter (82¼+⅔[⅒]). See Hogendijk, "Al-Qabīṣī's Treatise," 229 [Arabic], 201 [Engl. trans.]; cf. Istanbul, Süleymaniye Lib., MS Ayasofya MS 4832, f 94a. Purely speculating, this could be an inadvertent error, substituting 4½ for 4¼. In the *Planetary Hypo*thesis, Ptolemy's value for Saturn's diameter is 4¼+1/20, which produces a volume of 79½, a value more in line with the other scholars. See Goldstein, "The Arabic Version," 33 (BM MS Arab 426, f. 91b [Arabic]), 8–9 [Engl. trans.]).

73 Ptolemy does not give a volume for the smallest stars but does give a Hipparchan value for their apparent diameters (1/30 the size of the Sun) (Goldstein, "The Arabic Version," 31 [BM MS Arab 426, f. 90b (Arabic), 8 (Engl. trans.)]).

74 13;16 in the English translation is a misreading.

75 All four witnesses have double (*ḍiʿf*), rather than half (*niṣf*), which is clearly a copyist error.

76 The value of ≈ 1 15/49 (lit: 1 and 15 parts out of 49 and a small amount) is a computational error. Abū Maʿshar's true diameter for Mars compared to the Earth's (1⅐) was squared instead of cubed (see Ibn Rustah, 7:21 [lines 8–9]).

77 Kharaqī has 1;29′ (Ghalandari, *Muntahá*, 237).

Moon	1/39	1/40 [≈1/39¼]	1/39	≈1/39¼	1/40	1/39¼	1/39¼	1/39¼	0;1′,30″ [78]	1/39¼
Venus	1/44	1/44	1/37	1/44	1/37	1/44	≈1/36	1/44	0;1′,34″ [79]	1/34⅓
Mercury	1/22,000	1/19,683	≈1/22,000	1/22[000]	1/19,683	1/22,000	[1/18,088][80]	1/24,389	0;0,0,10,4 [0;0,0,10][81]	1/22,000

Table 3.

Relative and True Volumes (in Cubic Parasangs) According to Jaghmīnī

BODY[82]	Relative volumes (Earth as unit)	Jaghmīnī's true volumes in parasangs[83]	Modern calculated values using Earth's volume = 20,363,630⅓	Calculated values using Earth's volume = 20,363,636⅓ [suggested emendation]
Moon	1/39	522,145	20,363,630⅓÷39= 522,144.4	20,363,636⅓÷39= 522,144.5
Mercury	1/22,000	925	20,363,630⅓÷22,000= 925.6	20,363,636⅓÷22,000= 925.6
Venus	1/44	462,809	20,363,630⅓÷44= 462,809.8	20,363,636⅓÷44= 462,809.9
Sun	167	3,400,727,002⅓	20,363,630⅓x167= 3,400,726,265.7	20,363,636⅓x167= 3,400,727,267.7
Mars	[1½] [84]	30,5[4]5,[445]	20,363,630⅓x1.5= 30,545,445.5	20,363,636⅓x1.5= 30,545,454.5

78 0;1′,30″ = 1/40.

79 0;1′,34″ ≈1/38.

80 Battānī's value for Mercury's true diameter is 1/26¼ (approx.) of the Earth's, so the volume should be 1/18,087.89 or about 1/18,088 Earth volumes. However, the Arabic gives 1/17 (approx.). Nallino notes the error and replaces the 1/17 with 1/18087 in his Latin translation, also recording the value in Plato of Tivoli's translation as 1/19,000. See Nallino, 3:182 [Arabic], 1:121n7 [Latin trans.]).

81 0;0,0,10,4≈1/21,457. Petermann I MS 67, f. 33a has عشره واربعه روابع (i.e., 10 and 4 in the fourth [sexagesimal place]), which is the only manuscript I've seen that makes numerical sense. 0;0,0,10 = 1/21,600. *Muntahá* (236) has عشر روابع (i.e., 0;0,0,0,10), which is clearly incorrect.

82 The listing of the bodies is in accordance with Jaghmīnī's order in the text.

83 Digits in square brackets are reconstructed.

84 All four witnesses have double (*ḍiʿf*), rather than half (*niṣf*), which is clearly a copyist error.

Jupiter	82	[1,6]69,81[7],155	20,363,630⅓x82= 1,669,817,687.3	20,363,636⅓x82= 1,669,818,179.3
Saturn	80	1,629,090,883	20,363,630⅓x80= 1,629,090,426.7	20,363,636⅓x80= 1,629,090,906.7
15 [largest] fixed stars	95	1,000,545,420	20,363,630⅓x95= 1,934,544,881.7	20,363,636⅓x95= 1,934,545,451.7
Smallest [fixed stars]	15	305,454,045	20,363,630⅓x15= 305,454,455	20,363,636⅓x15= 305,454,545

Manuscripts Used for the Edition

The edited Arabic text is based upon the four extant manuscripts described below.[85] Each has deficiencies of one kind or another: mistakes in grammar or misreadings by the copyist; a missing section; or omitted parts of a parameter. On the other hand, none use the alphanumeric system, which lends itself to ambiguity and often introduces mistakes. That parameters were written out in words has proved valuable in establishing and/or confirming numerous values. In fact, among the four manuscript witnesses, relatively few variants are found, and these are given in the apparatus. Major variations are noted along with any comments in the footnotes to the English translation.

85 I am aware of four extant copies of this work: MS B is at the University Library of Bratislava in the Slovak Republic and was kindly brought to my attention by Sajjad Nikfahm-Khubravan; MS Q is located at the Egyptian National Library (Dār al-kutub); MSS K and N are both housed at the Süleymaniye Manuscript Library, Istanbul, Turkey. I am grateful to İhsan Fazlıoğlu and Elmin Aliyev for making me aware of the Süleymaniye manuscripts and facilitating access to them.

Sigla and Descriptions of the Manuscripts

<table>
<tr><th>Siglum</th><th>Description of Manuscript</th></tr>
<tr><td>1. ب [=B]</td><td>Bratislava, University Library of Bratislava, Bašagić Collection of Islamic Manuscripts, TG 15, Ordinal Number 291, f. 33a. The codex, of 361 pages, contains a collection of assorted treatises, including one listed as a work with no title by Maȟmūd bin Muȟammad bin ʿUmar al-Ĝaġmīnī. The work itself does not bear a date, but other works in the codex have a copy date of 987 H [=1579 CE]. See folio 29a and folio 62a as examples.

For the online description and image of this work, see: http://retrobib.ulib.sk/Basagic/EN/291.htm and for the entire codex, see: http://digitalna.kniznica.info/zoom/66996/view?search=%C4%9Ca%C4%A1m%C4%ABn%C4%AB&page=74&p=separate&tool=info&view=0,0,1773,2650

The introductory remarks in MS B are found with slight variations in the endings of MSS K, N, and Q. Part of the concluding text and the ending are written in the margin of MS B, and MS B lacks the additional section on measurement found in the three other manuscript witnesses.</td></tr>
<tr><td></td><td>Introductory Remarks and Incipit:
من فوائد الامام الچغمنى الخوارزمىّ حين فرغ من تأليف الملخص واهداه الي الامام بدر الدين الفلانسىّ(!) بسمِ الله الرحمن الرحيم الحمد لله خالق الاجرام والصلوة على نبيّه وآله واصحابه ما دارت اللىالى والايام قال رحمه الله ىعالى الشمس وما فوقها من الكواكب كل واحد منها اعظم من الارض وكل واحد من الكواكب تحت الشمس اصغر من الارض ...</td></tr>
<tr><td></td><td>From among the useful [writings] of the Imām al-Jaghmīnī al-Khwārizmī, [which he wrote] at the time he completed the composition of al-Mulakhkhaṣ, and he dedicated it to the Imām Badr al-Dīn al-Falānisī [!] In the Name of God, the Beneficent, the Merciful. Praise be to God, the Creator of the bodies, and may blessings be upon His Prophet, his family, and his companions as long as days and nights return in succession. [Al-Jaghmīnī], may God Almighty have mercy upon him, said: the Sun and the planets above it are each larger than the Earth, and each of the planets below the Sun is smaller than the Earth ...</td></tr>
<tr><td></td><td>Ending:
والله اعلم بالصواب واليه المرجع والمآب

And God is most knowing of the truth, and to Him are the refuge and the final return.</td></tr>
</table>

2. ق [=Q]

Cairo, Dār al-kutub, Ṭalʿat Majāmīʿ [ṬJ], MS 429 (2), f. 4a-4b. The Egyptian National Library catalogue lists MS Q as the second in a codex of assorted treatises; the only other scientific work listed for this codex (no. 14) is on the rainbow. According to the catalogue description, MS Q is a treatise on planetary distances and sizes dedicated to al-Imām Badr al-Dīn al-Falāsitī (?), written in a Fārsī hand, *ca.* 1100 H [=1689 CE], and possibly unique.[86] In fact, the work does not deal with planetary distances, al-Qalānisī has been misread, and this copy is not unique. Appended to the end of this witness is a citation from Qāḍīzāde's *Sharḥ al-Mulakhkhaṣ* commenting on the Earth's sphericity [see below for edition and translation].

Incipit:

بسم الله الرّحمن الرّحيم الحمد لوليه والصلوة على نبيه واله واصحابه العظام ما دامت الليالى والايام قال رحمه الله الشمس وما فوقها من الكواكب كل واجد(!) اعظم مِن الارض وكل واجد(!) من الكواكب تحت الشمس اصغر من الارض ...

In the Name of God, the Beneficent, the Merciful. Praise be to His friend, and blessings be upon His Prophet, his family, and his exalted companions as long as there are days and nights. [Al-Jaghmīnī], may God have mercy upon him, said: the Sun and the planets above it are each larger than the Earth, and each of the planets below the Sun is smaller than the Earth ...

Ending:

تمت الرسالة التى افادها الامام الجغمينى الخوارزمى حين فرغ من تاليف الملخص فى الهيئة واهداها الى الامام بدر الدىن الفلاستى (القلاستى؟) والله اعلم تم

The treatise is completed, which the Imām al-Jaghmīnī al-Khwārizmī put forth at the time he completed the composition of *al-Mulakhkhaṣ fī al-hayʾa*, and he dedicated it [them?] to the Imām Badr al-Dīn al-Falāsitī [sic], and God is all-knowing.

86 See D.A. King, *A Survey*, 150 (G17, 1.2.7) and King, *Fihris*, 1:556 and 2:21 [in Arabic].

3. ك [=K]	**Istanbul, Süleymaniye Manuscript Library, Kasidecizade MS 710, ff. 131b-132a.** The codex is a compilation of a huge number of works, in various hands. MS K begins on f. 131b with the no. 69 written in the margin, indicating that it is the sixty-ninth work in this section of the codex. (There are numerous other sections in different hands.) The work is titled a *hayʾa* treatise by the Imām al-Jaghmīnī.
	Title and incipit:
	رسالة من الهيئة بسم الله الرّحمن الرّحيم للامام الجغمني الحمد لله حالق الاجرام والصلوة على نبيه وآله وصحبه العظام ما دامت الليالي والايام قال الشمس وما فوقها من الكواكب كل واحد منها اعظم من الارض وكل واحد من الكواكب تحت الشمس اصغر من الارض …
	A *hayʾa* treatise. In the Name of God, the Beneficent, the Merciful. By the Imām al-Jaghmīnī. Praise be to God, the Creator of the bodies, and may blessings be upon His Prophet, his family, and his exalted companions as long as there are days and nights. [Al-Jaghmīnī] said: the Sun and the planets above it are each larger than the Earth, and each of the planets below the Sun is smaller than the Earth …
	Ending followed by the section on measurement:
	والله اعلم بالصواب من فوايد الامام الجعمنى الخوارزمي كتبها حين فرغ من تاليف الملخص واهداها الى الامام بدر الدّين القلانسي And God is most knowing of the truth. From among the useful [writings] of the Imām al-Jaghmīnī al-Khwārizmī, which he wrote at the time he completed the composition of *al-Mulakhkhaṣ*, and he dedicated it [them?] to the Imām Badr al-Dīn al-Qalānisī.

4. ن [=N]	**Istanbul, Süleymaniye Manuscript Library, Mehmet Nuri Effendi MS 197, f. 12a-12b.** The codex contains 194 folios. A table of contents lists 71 works, including MS N, titling it a treatise on *al-hayʾa* by the Imām al-Jaghmīnī. Evidently, whoever compiled the table of contents was more aware of al-Jaghmīnī than the copyist, who refers to him as "al-Jaghmīn" twice.
	Title (in red ink) and incipit: رسالة الهيـة للامام الجعمين رحمه اللـــه تعالى بسم الله الرحمن الرحيم الحمد لله حالق الاجرام والصلوة على نبيه واله واصحابه العظام ما دامت الليالى والايام قال الشمس وما فيها(!) الكواكب كا(!) واحد منها اعظم من الارض وكل واحد من الكواكب تحت الشمس اصغر من الارض …
	A *hay'a* treatise by the Imām al-Jaghmīn [sic], may God Almighty have mercy upon him. In the Name of God, the Beneficent, the Merciful. Praise be to God, the Creator of the bodies, and may blessings be upon His Prophet, his family, and his exalted companions as long as there are days and nights. [Al-Jaghmīnī] said: the Sun and what is in it [sic] the planets are each larger than the Earth, and each of the planets below the Sun is smaller than the Earth …
	Ending followed by title (partially overlined in red [in bold here]) beginning the final section: والله اعلم بالصواب **من فوائد الامام الجعمين الخوارزمى كتبها حين فرغ من تاليف الملخص واهداها الى الامام** بدر الدين القلانسى And God is most knowing of the truth. **From among the useful [writings] of the Imām al-Jaghmīn [sic] al-Khwārizmī**, which he wrote at the time he completed the composition of *al-Mulakhkhaṣ*, and he dedicated it [them?] to the Imām Badr al-Dīn al-Qalānisī.

Text and Apparatus Conventions

Text Conventions

1. The orthography and rules for writing *hamza*s, numbers, and numerals follow modern conventions; divergences are not noted except where alternative readings might occur (such as between *thulth* and *thalāth*). When giving variants, I have written these as they are found in the text, providing or leaving out the dots, vowels, and *hamzas* as given.

2. The dotting of *yāʾ* follows the rules used by printers in Syria and Lebanon.

3. *Tanwīn* is generally added (but not to feminine *tāʾ* endings).

4. *Shaddas* have been supplied (except for sun letters and *nisbas*).

5. Short vowels have been provided sparingly as aids to the reader and/or to avoid ambiguity.

Apparatus Conventions

[	Separates the reading in the edition from any variant
:	Separates the variant and the manuscript *sigla*
+	Added in
–	Missing from
=	Indicates another variant
(…)	Editor's comments

Edition

[B]ب : Bratislava, University Library, Bašagić Collection, TG 15, Ordinal Number 291, f. 33a

[Q]ق : Cairo, Dār al-kutub, Ṭalʿat Majāmīʿ MS 429, f. 4a–4b

[K]ك : Istanbul, Süleymaniye Manuscript Library, Kasidecizade MS 710, ff. 131b-132a

[N]ن : Istanbul, Süleymaniye Manuscript Library, Mehmet Nuri Effendi MS 197, f. 12a–12b

[1]من فوائد الإمام الجغميني الخوارزمي حين فرغ من تأليف الملخّص وأهداه إلى الإمام بدر الدين القلانسي[2]

بسم الله الرحمٰن الرحيم

الحمد لله خالق الأجرام[3] والصلوة على نبيه وآله وأصحابه العظام[4] ما دامت[5] الليالي والأيّام قال رحمه الله[6] الشمس وما فوقها من[7] الكواكب كلّ واحد منها أعظم[8] من الأرض وكلّ واحد[9] من الكواكب تحت الشمس أصغر من الأرض.

فالشمس مائة وسبعة[10] وستّون ضعفاً للأرض؛
والمرّيخ مثل وضعفه(!)؛[11]
والمشتري اثنان وثمانون ضعفاً؛
وزحل ثمانون.[12]

ومن الثوابت خمسة عشر كوكباً كلّ واحد منها خمسة وتسعون ضعفاً ثمّ ينقص قليلاً إلى أن يكون أصغرها خمسة عشر مثلاً للأرض.

١ ٣٣ آ: ب = ٤ آ: ق = ١٣١ب: ك = ١٢ آ: ن.

٢ من فوائد الإمام الجغميني الخوارزمي حين فرغ من تأليف الملخّص وأهداه إلى الإمام بدر الدين القلانسي] من فوائد الامام الچغمنى الخوارزميّ حين فرغ من تأليف الملخص واهداه الي الامام بدر الدين الفلاسىّ: ب = —ق = رسالة من الهيئة بسم الله الرّحمن الرّحيم للامام الچغمني: ك = رسالة الهيئة للامام الچعمين رحمه الله تعالى: ن.

٣ الحمد لله خالق الأجرام] ب، ك، ن = الحمد لوليه: ق.

٤ وأصحابه العظام] ق، ن = واصحابه: ب = وصحبه العطام: ك.

٥ ما دامت] ق، ك، ن = ما دارت: ب.

٦ رحمه الله] ق = +تعالى: ب = —ك، —ن.

٧ وما فوقها من] ب، ق، ك = وما فيها: ن.

٨ كلّ واحد منها أعظم] ب، ك = كل واجد اعظم: ق = كا واحد منها اعظم: ن.

٩ وكلّ واحد] ب، ك، ن = وكل واجد: ق.

١٠ وسبعة] وسبع: ب، ق، ك، ن.

١١ مثل وضعفه] مثل وضعف: ب، ق، ك = مثلا وضعفه: ن.

١٢ وزحل ثمانون] ب، ق، ك = وزخل ثمان: ن.

وأمّا ما تحت الشمس:
فالأرض أربعة وأربعون ضعفاً للزهرة؛[١٣]
واثنان وعشرون ألف ضعف لعطارد؛
وتسعة[١٤] وثلاثون ضعفاً للقمر.

فأعظم الأجرام الشمس ثمّ الثوابت العظيمة الخمسة عشر[١٥] ثمّ المشتري ثمّ زحل[١٦] ثمّ الثوابت الصغيرة بمراتبها ثمّ المرّيخ ثمّ الأرض ثمّ القمر ثمّ الزهرة ثمّ عطارد.[١٧]

وأمّا تفصيل مقادير الأجرام:[١٨]
فجرم الأرض عشرون ألف ألف وثلاثمائة وثلاثة[١٩] [٢٠]وستّون ألفاً[٢١] وستّمائة وثلاثون فرسخاً وثُلث[٢٢] فرسخ بالمقدار الذي هو فرسخ في فرسخ في فرسخ.[٢٣]

وبهذا المقدار أيضاً:
جرم القمر خمسمائة ألف واثنان وعشرون ألفاً ومائة وخمسة[٢٤] وأربعون؛
وجرم عطارد تسعمائة وخمسة وعشرون؛
وجرم الزهرة أربعمائة ألف واثنان وستّون[٢٥] ألفاً وثمانمائة وتسعة؛

١٣ للزهرة] ب، ك، ن = للز ة: ق.
١٤ وتسعة] وتسع: ب، ق، ك، ن.
١٥ الخمسة عشر] ب، ك، ن = خمسة عشر: ق.
١٦ زحل] ب، ق، ك = زخل: ن.
١٧ ثمّ عطارد] ثم عطار: ب = —ق، —ك، —ن.
١٨ الأجرام] ق، ك، ن = اجرامها: ب.
١٩ وثلاثمائة وثلاثة] وثلثمائه وثلثه: ب = وثلثمائه: ق = وثلثمائة وثلثة: ك، ن.
٢٠ ١٣٢ آ: ك.
٢١ ألفاً] الف: ب، ق، ك، ن.
٢٢ وثُلث] وثلثه: ب، ق، ك = وثلمه: ن.
٢٣ هو فرسخ في فرسخ في فرسخ] ب، ق، ك = هو فرسخ فى فرسخ: ن.
٢٤ ١٢ب: ن («وخمسة» مكرّرة على الصفحة التالية).
٢٥ أربعمائة ألف واثنان وستّون] (مكرّرة في ق).

وجرم الشمس ثلاثة آلاف ألف ألف وأربعمائة ألف ألف وسبعمائة وسبعة وعشرون ألفاً وفرسخان وثُلث فرسخ؛

وجرم المرّيخ ثلاثون ألف ألف وخمسمائة وخمسة

وتسعة وستّون[٢٦] ألف ألف وثمانمائة[٢٧] عشر ألفاً[٢٨] ومائة وخمسة وخمسون؛

وجرم زحل[٢٩] ألف ألف ألف وستّمائة وتسعة وعشرون ألف ألف[٣٠] وتسعون ألفاً[٣١] وثمانمائة وثلاثة وثمانون؛

وجرم كلّ واحد من الثوابت[٣٢] الخمسة عشر ألف ألف وخمسمائة وخمسة[٣٣] وأربعون ألفاً[٣٤] وأربعمائة وعشرون؛

وجرم كلّ واحد من أصغرها ثلاثمائة ألف ألف وخمسة آلاف ألف وأربعمائة وأربعة وخمسون ألفاً وخمسة[٣٥] وأربعون.[٣٦]

وجميع هذا بالمقدار الذي هو فرسخ في فرسخ في فرسخ.[٣٧]

٢٦ وخمسة وتسعة وستّون] ب، ك، ن = وتسعة وتسعون: ق.

٢٧ وثمانمائة] ق = او ثمانمائة: ب، ك، ن.

٢٨ ألفاً] ب = الف: ق، ك، ن.

٢٩ زحل] ب، ق، ك = زخل: ن.

٣٠ ٤ب: ق.

٣١ ألفاً] ب، ق = الف: ك، ن.

٣٢ الثوابت] ب، ك، ن = الكواكب: ق.

٣٣ وخمسة] —ن.

٣٤ ألفاً] ب، ك، ن = الف: ق.

٣٥ وخمسة] وخمسا: ب، ق = وخمس: ك، ن.

٣٦ وأربعون] ب، ق، ك = واربعو: ن.

٣٧ وخمسمائة وخمسة وأربعون ألفاً ... هو فرسخ في فرسخ في فرسخ] (في هامش ب) = +والله اعلم بالصواب واليه المرجع والمآب: (في هامش ب) = +والله اعلم بالصواب من فوايد الامام الجعمنى الخوارزمي كتبها حين فرغ من تاليف الملخص واهداها الى الامام بدر الدّين القلانسي: ك = +والله اعلم بالصواب من فوائد الامام الجعمين الخوارزمى كتبها حين فرغ من تاليف الملخص واهداها الى الامام بدر الدين القلانسى: ن (يوجد خط أحمر فوق السطر من «من فوائد» إلى «واهداها الى الامام»).

[ملحق مخطوطات ق، ك، ن]

الفرسخ في اصطلاحهم ثلاثة أميال بالاتّفاق وأمّا الميل فهو عند المتقدّمين ثلاثة آلاف ذرعان(!)[٣٨] وعند المتأخّرين أربعة آلاف[٣٩] وليس ذلك للاختلاف[٤٠] في الميل بل في الذراع لأن الذراع إذا كان عند المتأخّرين أربعة وعشرين[٤١] إصبعاً وعند المتقدّمين اثنين[٤٢] وثلاثين كما سنذكره[٤٣] كان الميل على التفسيرين واحداً وهو ستّة وتسعون[٤٤] ألف إصبع فلا اختلاف في تفسير الفرسخ ولا الميل بين[٤٥] المتقدّمين والمتأخّرين وإن توهّمه[٤٦] بعضهم نظراً إلى التفسيرين المذكورين للميل[٤٧]. سنان باشا لقاضي زاده[٤٨]

[خاتمة مخطوط ق]

تمّت الرسالة التي أفادها الإمام الجغميني الخوارزمي حين فرغ من تأليف الملخّص في الهيئة وأهداها إلى الإمام بدر الدين القلانسي[٤٩] والله أعلم تمّ

[ملحق مخطوط ق]

اعلم أنّ قُطر الأرض على ما وجده المتقدّمون ألفان وخمسمائة وخمسة وأربعون فرسخاً تقريباً وأنّ أعظم الجبال فرسخان وثُلث فرسخ وهو خمسة أمثال لنصف[٥٠] فرسخ تقريباً. قاضي زاده رحمه

٣٨ ذرعان] ذراع: ق، ك، ن.

٣٩ أربعة آلاف] +ذراع: ق، ك، ن.

٤٠ للاختلاف] الاختلاف: ق، ك، ن.

٤١ وعشرين] وعشرون: ق، ك، ن.

٤٢ اثنين] باثنين: ق، ك، ن.

٤٣ سنذكره] ذكره: ق، ك، ن.

٤٤ وتسعون] ن = وسبعون: ق.

٤٥ بين] (غير مقروء في ق).

٤٦ توهّمه] ق، ك = يهمه: ن.

٤٧ المذكورين للميل] بالميل: ق، ك، ن.

٤٨ سنان باشا لقاضي زاده] ك = —ق، —ن.

٤٩ القلانسي] الفلاستى (القلاستى؟): ق.

٥٠ أمثال لنصف] اميال بنصف: ق.

Translation

[87] From among the useful [writings] by the Imām al-Jaghmīnī al-Khwārizmī, [which he wrote] at the time he completed the composition of *al-Mulakhkhaṣ*, and he dedicated it to the Imām Badr al-Dīn al-Qalānisī [88]

In the Name of God, the Beneficent, the Merciful

Praise be to God, the Creator of the bodies, and may blessings be upon His Prophet, his family, and his exalted companions as long as there are days and nights. [Al-Jaghmīnī], may God have mercy upon him, said: the Sun and the planets above it are each larger than the Earth, and each of the planets below the Sun is smaller than the Earth.

The Sun is 167 times [the size of] the Earth;
Mars is equal [to the Earth] and double [sic] it[89];
Jupiter is 82 times;
Saturn is 80 [times].

Among the fixed stars, there are 15 stars (*kawkab*)[90], each of which is 95 times [the Earth]; they then decrease incrementally until the smallest of them is 15 times the Earth.

As for what is below the Sun:
The Earth is 44 times [the size of] Venus;
22,000 times Mercury;
and 39 times the Moon.

So, the largest of the bodies is the Sun, then the 15 large, fixed stars, then Jupiter, then Saturn, then the small[er] fixed stars according to their rank, then Mars, then the Earth, then the Moon, then Venus, then Mercury.

87 Folio 33a: B = folio 4a: Q = folio 12a: N.

88 These are the introductory remarks in MS B; they are also found (with slight variations) in the explicit of MS Q. MS N (in red ink) has the title "A *hayʾa* treatise by the Imām al-Jaghmīn [sic], may God Almighty have mercy upon him."

89 All four witnesses have double (*ḍiʿf*), rather than half (*niṣf*), which is clearly a copyist error. One and a half times the Earth is the Ptolemaic value for Mars.

90 One possible source for the specific number of 15 large, fixed stars is Kharaqī's *Muntahā,* bk. I, ch. 12 (On the Configuration of the Fixed Stars) (Ghalandari, 77 [147]).

As for detailing the sizes of the bodies:

the Earth's body [i.e., volume] is 20,363,630⅓ parasangs,[91] this being in the measure that is parasang times parasang times parasang [i.e., cubic parasangs].[92]

Using the same measure:
the volume (*jirm*) of the Moon is 522,145;[93]
the volume of Mercury is 925;[94]
the volume of Venus is 462,809;[95]
the volume of the Sun is 3,400,727,002⅓ parasangs;[96]
the volume of Mars is 30,5[4]5,[445];[97]
[the volume of Jupiter is 1,6]69,81[7],155;[98]
the volume of Saturn is 1,629,090,883;[99]
the volume of each one of the 15 [largest] fixed stars is 1,000,545,420;[100]
and the volume of each one of the smallest [fixed stars] is 305,454,045.[101]

91 Kharaqī cites the calculated Ptolemaic value of 183,264,000 sq. miles for the Earth's surface (Ghalandari, *Muhtahá,* 231 [417]), which is about 20,362,666⅔ sq. parasangs. Al-Ṭūsī also gives 183,264,000 miles in his Persian *al-Risāla al-Muʿīniyya* (see 176, IV.1[2]).

92 Note that Jaghmīnī takes the 20,363,630⅓ to be a cubed value (i.e., for volume). But as mentioned above, Kharaqī and Ṭūsī give a similar value (albeit in miles) for the Earth's surface area (i.e., in square parasangs), not its volume (i.e., in cubic parasangs). That this is not a copyist error is confirmed, inasmuch as it is used subsequently for the planetary and stellar volumes derived from Jaghmīnī's incorrect value for the Earth's volume. All astronomers that I have checked who lived after Jaghmīnī were fully aware that this parameter is the Earth's surface area (i.e., not its volume); see Table 1, fn. 50.

93 20,363,630⅓ ÷ 39 = 522,144.4

94 20,363,630⅓ ÷ 22,000 = 925.6

95 20,363,630⅓ ÷ 44 = 462,809.8

96 20,363,630⅓ x 167 = 3,400,726,265.7. I don't know the reason for the discrepancy, whether due to a calculation error or scribal miscopying.

97 20,363,630⅓ x 1.5 = 30,545,445.5. The 445 is completely missing as is the 4 in the ten-thousands place. But enough of the number is extant to confirm the computation, at least to the thousands place.

98 20,363,630⅓ x 82 = 1,669,817,687.3. Clearly there is considerable corruption for Jupiter. For one thing, the name Jupiter is missing, as well as the first part of the number up to the hundred-millions place. But the rest of the number, with the exception of 7 in the thousands place, is present. The discrepancy between 687 and 155 could just be a calculation error. The rest of the corruption is probably scribal, due to the conflation with the first part of Saturn's number.

99 20,363,630⅓ x 80 = 1,629,090,426.7. Again, I don't know the reason for the discrepancy in the hundreds, tens and units places, whether due to a calculation error or scribal miscopying.

100 20,363,630⅓ x 95 = 1,934,544,881.7. Apparently the 934 has been dropped along the way; as for the other discrepancies, they again might be attributable to calculation errors or scribal miscopying.

101 20,363,630⅓ x 15 = 305,454,455. The small differences in the final digits may either be due to calculation errors or, more likely in this case, scribal corruption.

All these are in the measure that is parasang times parasang times parasang [i.e., cubic parasangs].[102]

[Appended to MSS K, N, and Q][103]

[104]According to their agreed convention, the parasang is three miles. Regarding the mile, for the Ancients it was 3,000 cubits, and for the Moderns 4,000 cubits. That difference is not in the mile but rather in the cubit, because inasmuch as the cubit for the Moderns is 24 digits and for the Ancients it is 32 [digits],[105] as we shall point out, the mile is the same according to the two accounts, namely 96,000 digits.[106] Thus there is no difference between the Ancients and the Moderns in defining either the parasang or the mile, even though some of them imagined it to be so on the basis of the two above-mentioned accounts for the mile. Sinān Pāshā on Qāḍīzāde

[Ending of MS Q]

The treatise is completed, which the Imām al-Jaghmīnī al-Khwārizmī put forth at the time he completed the composition of *al-Mulakhkhaṣ fī al-hayʾa*, and he dedicated it [them?] to the Imām Badr al-Dīn al-Falāsitī [sic], and God is all-knowing.[107]

102 MS B ends here with the following: "And God is most knowing of the truth, and to Him are the refuge and the final return." MSS K and N also have what purports to be an ending: "And God is most knowing of the truth." However, both then add the following, (partially overlined in red ink in MS N), before beginning the final section: "From among the useful [writings] of the Imām al-Jaghmīnī [al-Jaghmīn in MS N] al-Khwārizmī, which he wrote at the time he completed the composition of the work *al-Mulakhkhaṣ*, and he dedicated it [them?] to the Imām Badr al-Dīn al-Qalānisī." For the Arabic, see variant 37.

103 This is a direct citation from Sinān Pāshā's *Ḥāshiya* on Qāḍīzāde's *Sharḥ al-Mulakhkhaṣ* on the parasang. Sinān Pāshā (d. 891 H/1486 CE) dedicated the *Gloss* to Bayezid II (r. 886-918 H/1481-1512 CE). For the Arabic text, I used Istanbul, Topkapı Sarayı Müzesi, Ahmet III MS 3299 (f. 12a); it is the presentation copy from Bayezid II's library and bears the Sultan's seal. I am indebted to Mehmet Arıkan for obtaining a copy of this witness for me. Variants from the manuscripts used for the edition are noted in the footnotes.

104 This final section on the parasang by Sinān Pāshā is in MSS K, N and Q; it is missing in MS B.

105 For the Ptolemaic standard value for digits, both Kharaqī and Kūshyār ibn Labbān stated that 1 cubit = 36 digits, not 32 digits. For Kharaqī, see Ghalandari, *Muhtahá*, 231 [416]; cf. Berlin, Staatsbibliothek, Landberg MS 33, f. 49b. For Kūshyār, see his *Zīj al-Jāmiʿ*, bk. III, ch. 22 (Leiden, Univ. Library MS Or 8, f. 99b) and his *Risāla* (Bagheri, Hogendijk, and Yano, "Kūshyār," 107 [Arabic], and 82 [Engl. trans.]).

106 Clearly 32 (i.e., not 36) is the correct number of digits in order to have an equivalence of total number of digits [96,000] per mile for both the Ancients and the Moderns: for the Ancients, 3,000 cubits x 32 digits = 96,000, and for the Moderns, 4,000 cubits x 24 digits = 96,000. See also, F.J. Ragep, *Ṭūsī's Memoir*, 2:508–9, esp. 508n32 and the introductory section above "On Measurement."

107 Note that all four manuscripts have a similar statement about the text but in different places: MS B has

[Appended to MS Q][108]

Know that the Earth's diameter, according to what the Ancients found, is approximately 2,545 parasangs and that the [height of the] greatest mountain is 2⅓ parasangs, approximately five times[109] half a parasang. Qāḍīzāde, may God have mercy upon him.

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it at the beginning, MSS K and N before the final section, and MS Q at the end.

108 I have corrected the copyist mistakes in both the edition and translation, since the note is a direct citation from Qāḍīzāde's *Sharḥ al-Mulakhkhaṣ* [see, for example, Istanbul, Süleymaniye, Ayasofya MS 2662, f. 6a]. Qāḍīzāde is commenting on Jaghmīnī's passage dealing with the Earth's sphericity, and how the shape is not affected by undulations that occur on the Earth's surface due to mountains and valleys. See S.P. Ragep, *Jaghmīnī's Mulakhkhaṣ*, 86–87, 179 (Intro. [1]).

109 The copyist of MS Q has miles (*amyāl*) instead of times (*amthāl*).

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