# A Mathematical Analysis of the Theory of Horizontal Sundials in the Ottoman Period: The Case of *Risālah of Ruhāma*

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NAZARİMAT 🗱

**Abstract:** As the prayer times are determined according the sun's position, the theory of sundials, how to draw and design them, hold a significant place in the history of Islamic astronomy. All Muslim scientists working on applied and theoretical sciences have books on this subject. Besides, as it was a part of *madrasa* curriculum, there are considerably high number of unexamined books from the beginning to the end of the Ottoman period. This study is about a manuscript titled *Risālah-i Vaż' i Ruhāma*, that we received from a second-hand book seller. Its anonymous author argues that he offers a simpler way to draw sundials designed for Istanbul. The author leaves the person willing to make a sundial free to determine the height of the gnomon and he shows a practical method to draw ruhāma with the help of goniometer and ruler prepared according to the determined height. Although the article presents the mathematical bases of the method, it is unable to verify clearly how the author derived the numeric values given in the book. To the end of our study, facsimile text and its simplified transcription are attached.

**Keywords:** Theory of Sundials, gnomonic, Muslim Sundials, horizontal sundials, time systems in Islam, prayer times, *ruhāma*.

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#### 1. Introductio

he time period of the Sun virtually revolving around the world is defined as 1 *full day*. In ancient Egypt – without taking the seasons into account – the days and nights, independent of each other, are divided into 12 equal hours. This conception of time is labeled as *seasonal* or *temporal* hour. On the 41<sup>st</sup> latitude, where Istanbul is located, on the 22<sup>nd</sup> of June, the longest day is 16 hours and 22 minutes, and the shortest night is 7 hours and 38 minutes; and on the 22<sup>nd</sup> of December, the shortest day is 9 hours and 27 minutes, whereas the longest night is 14 hours and 33 minutes. If these time lengths are divided into 12, in the summers the longest day hour is 1 hour and 21.83 minutes, and the shortest night hour is 38.17 minutes; and in the winters the shortest day hour is 47.25 minutes, and the longest night hour is 1 hour and 12.75 minutes. On the 21<sup>st</sup> of March in the spring, and 23<sup>rd</sup> of September in the autumn, on the other hand, the time lengths of day and night are equal and each is 12 hours.<sup>1</sup>

In ancient Egypt, and in the ancient world, time was determined during the daylight by measuring the shade-length of a stick (gnomon) fixed into the ground. For example, the obelisk in Sultan Ahmet Square in Istanbul, which was brought to the city in the 4<sup>th</sup> century from the Temple of Ammon in Karnak/Egypt, was a gnomon of a sundial. The hours in the night used to be determined by the rising times of certain stars or by their passage through the longitude. In addition, sandglasses and water clocks were also used as auxiliary methods to supplement day time calculation. In sandglasses, time is measured by the flow of sand in a bottle, and in water clocks, the flow of the water quantity is measured in specific time intervals. Additionally, people used candle clocks at night.

In Mesopotamia, days are divided into 12 hours of equal length. Herodotus (490-425 BC) narrates that the Greeks learned about sundials and dividing days into 12 from Babylon. In the Hellenistic period (330 BC -30), days began to be divided into 24 equal hours, as it is the case today, and especially as it is like seasonal hours of March  $21^{st}$  and September  $23^{rd}$ . Such kind of hours are called equal hours. Yet, taking midday or midnight as the beginning of the new calendar day (*alafranga, zawālī*), and taking sunset as the beginning of new day (*alaturka, ezānī*) are two different applications of this time measurement.

In the Lunar calendar or Hijri calendar, Lunar months begin with the sighting of the Moon as a thin crescent during sunset. The new month begins the new cal-

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endar day, and when the Sun's top edge is tangent to the horizon during sunset, the hour is accepted to be as 12:00 or 0:00. The night that begins is considered not to be part of the previous day but the new one. As a result of this relatively defined day, the time span between two consecutive nights is not exactly equal to 24 hours. In the springs, it is a little longer, and in the autumns it is a little shorter. Hence, mechanical clocks need to be set to 12 manually every day during sunset.

The moment when the center of the Sun is just on the circle of the longitude is defined as midday. Timepieces which accept that moment as 12:00 are called noon-related *alafranga* or *zawālī* hour times. The word *zawāl* means exact midday time. To prevent possible confusions that may stem from setting the beginning of a new day at midday, usually in this hour system - as it is the case today - the day begins 12 hours before the midday, which means midnight.<sup>2</sup>

Since fasting and five daily prayers are ordered by Islam, and since these deeds are closely related to specific time slots, Muslims paid considerable attention to determining the times of noon, mid- afternoon and sunrise. In addition to this characteristic which we do not observe in religions other than Islam, accumulation of experience about this subject led to the production of more exact and more accurate timepieces, which in the past used to be produced just to know the time of the day. Besides, we do not know use of any devices for determining time, by Arabs in the pre-Islamic era, neither do we know their presence at the time of the Prophet Muhammad and four caliphs who succeeded him.

The first works about horizontal sundials are written in the Abbasid period. Although sources they used for calculating and drawing sundials were from pre-Islamic civilizations such as Ancient Greek, Egyptian and Indian civilizations, Muslim astronomers made considerable theoretical and practical contributions to the development of sundials. First Muslim astronomers who wrote on sundials are Ibrāhīm al-Fazārī (2<sup>nd</sup>/8<sup>th</sup> century), Habash al- Ḥāsib (3<sup>rd</sup>/9<sup>th</sup> century), Muḥammad ibn Mūsā al-Khwārizmī (2<sup>nd</sup>-3<sup>rd</sup>/8-9<sup>th</sup> century), Muḥammad ibn Ṣabbāh (3<sup>rd</sup>/9<sup>th</sup> century), al-Farghānī (3<sup>rd</sup>/9<sup>th</sup> century), Ibn al- 'Adamī (3<sup>rd</sup>/9th century) and Abū 'Abd Allah Muḥammad ibn al-Ḥasan ibn Abī Hishām al- Shatawī (3<sup>rd</sup>/9-10<sup>th</sup> century).<sup>3</sup> To our knowledge, the first book written in Islamic civilization about sundials is the al- Fazārī's *Kitāb al- mikyās li al- zawāl*, which he wrote when he was in Baghdad, but unfortunately is lost.<sup>4</sup> The oldest extant book on Islamic sundial is '*Amal al- sāʿa fī* 

3 D. A. King, "Mizwala", El<sub>2</sub>, c. 7, 115-16; D. A. King, Astronomy in the Service of Islam (Londra: Variorum Reprints, 1993), c. VIII; D. A. King, "Astronomical Instrumentation in the Medieval Near East", Islamic Astronomical Instruments içinde (Londra: Variorum Reprints, 1987), c. I.

<sup>2</sup> Bir, Kaçar, Acar, *Güneş Saatleri Yapım Kılavuzu*, pp. 29-30.

<sup>4</sup> Boris A. Rosenfeld, "Sundails in Islam", *Encyclopaedia of the History of Science, Technology, and Medicine in Non-Western Cultures*, ed. H. Selin (Boston & London: Kluwer Academic Pub. Dordrect), pp. 921-922.

*basīț al- ruhāma*, which is written in the early 9<sup>th</sup> century in Baghdad and attributed to al-Khwārizmī. The most important work which clearly explains how different sundials known in the Islamic world are produced is Thābit ibn Qurra's *Kitāb fī ālāt al- sā'a allatī tusammā ruhāmāt*.<sup>5</sup> The book is at the Koprulu Library in Istanbul.<sup>6</sup>

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Picture 1. The cover page of Thābit ibn Qurra's *Kitāb fī ālāt al-sāʿa allatī tusammā ruhāmāt* (Koprulu Library 984/1)

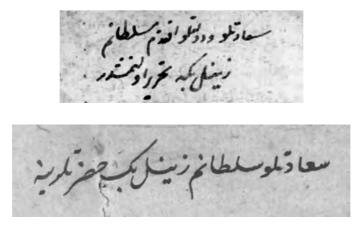
The manuscript entitled *Risālah-i vaż*'-*i ruhāma* (*Book on Ruhame Construction*) that we analyze in this article is the first known work written in the Ottoman period, which discuss Islamic sundials. Its author is unknown. On the inside of its cover the following phrase is seen: "Written for his Excellency Zeynel Beg" (Picture 2-a). Zeynel Beg (d. 1589) is the grandson of Asad al- dīn Kalanī who founded the Hakkari Emirate. He was appointed as the Amir of Hakkari by the Ottoman state, and supported by Süleyman I (1520-1566), Selim II (1566-1574) and Murad III (1574-1595). He was martyred in Merent during the Tabriz campaign; and after the conquest of the city, his corpse was brought to Hakkari (it was called Çölemerik at the time), and buried in the courtyard of Zeynel Beg Madrasa which was built by him.<sup>7</sup> There is a very similar note "to his excellency Zeynel Beg", which gives the

<sup>5</sup> Regis Morelon, "Thabit b. Qurra and Arab Astronomy in the 9th Century", Arabic Sciences and Philosophy, A historical journal, ed. R. Rashed, c. 4 (Cambridge, 1994), 111-139; Pouyan Rezvani, Three Treatises by Thâbit ibn Qurrah (Tahran, 2013), pp. 3-4.

<sup>6</sup> Köprülü Library, nr. 984/1.

<sup>7 &</sup>quot;Zeynel Beg was a loyal and brave one among Kurdish Begs in Hakkari [a city located in southeastern

impression of being written by the same person, in a corpus (majmuʻa) at Topkapi Palace Library, in the collection of Hazine (n. 452), which includes writings on astronomy and astrology about Taqī al- dīn al- Rāṣid and Aḥmad-i Dāʻī (Picture 2-b). The 17.5x11 cm sized hard-covered book that contains also this work includes three different works. The first one is an astronomy book about stars, and the third one is an astrology book dated 890/1485.



**Picture 2** (a) The phrase from the inside of the cover of the manuscript which states it is written for Zeynel Bag (above), (b) A similar dedication phrase from the manuscript at Topkapı Palace Library Hazine number 452 (below).

Though in the manuscript it is argued that drawing sundial is very easy and absolutely accurate, the text does not mention the kind of sundial. While it is stated that the sundial is designed for Istanbul ( $\varphi = 41^{\circ}$ ), the text does not give information about the azimuth angle of the wall which is required for the vertical sundials. The fact that the values about both kind of *ghurūbī* hour systems (*Babylonian* and *italic*) that are used in the drawing, with *zawālī* hour system, are given symmetric to the meridian, and more especially that the equinoctial line is given horizontal indicate that this timepiece is a horizontal sundial. In addition, the fact that drawing the *asr-i awwal* and *asr-i thāni* curves on the same plane with both *ghurūbī* hour system supports this assumption. Since wall clocks are usually drawn directly onto the wall, the name *ruhāma*, literally means marble, given to this timepiece should refer to a horizontal sundial.<sup>8</sup>

erence to the text itself. We will present our opinions about that drawing at the end of the article.

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part of Turkey], who lived in the time of Süleyman I." (*Sicilli-Osmani* 5, Tarih Vakfi Yurt Yayınları, 1996, 1709). (translation made by the authors) There is a sundail drawing at the end of the text, but there is nothing on the drawing which makes ref-

<sup>8</sup> 

#### 2. Mathematical Analysis

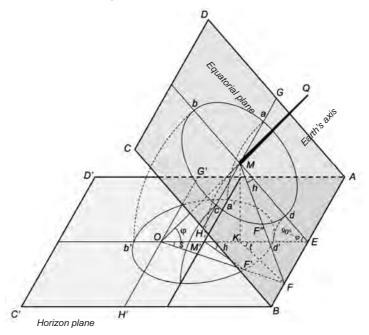
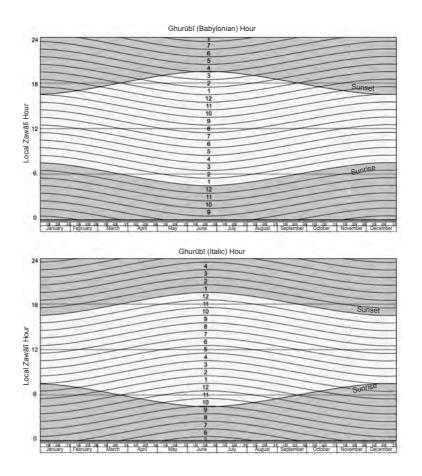


Figure 1. The relation between equatorial sundial and horizontal sundial

Since the quadrant of an equatorial sundial is parallel to the equatorial plane, a horizontal sundial can be thought as the projection of it on the horizontal plane. As can be seen in Figure 1, the stick of the equatorial sundial, which is parallel to the universe axis (polos) intersects the equatorial sundial at the point M. The radial MF hour lines that have  $15^{\circ}$  angle difference from each other are around this point.  $Zaw\bar{a}l\bar{i}$  hour system is obtained if the hour lines begins from ME meridian, whereas it is defined as ghurubi hour system if they begins from MG line<sup>9</sup> (of sunsie) or from MH line<sup>10</sup> (of sunset). The diagram in Figure 2 shows the change in the beginning of hours between *zawalī* hour system and both of *ghurūbī* hour systems throughout the year in Istanbul. Accordingly, while the *zawālī* system shows the time remaining to the midday or the time elapsed after it, *ghurūbī* system shows the time elapsed after sunrise or the time remaining to the sunset.

<sup>9</sup> This hour system originated from Mesopotamia is called Babylon hour system.

<sup>10</sup> Due to the fact that the Italiens first started to use this hour system, after having been introduced in Islamic World, it is called, in other European countries, the Italic hour system.



**Figure 2.** Diagrams for the change between *zawālī* and *ghurūbī* hour systems in a year on the latitude of Istanbul.

#### 2.1. Deriving Equations of Hour Angles and Lines

If we turn the *ABCD* plane that is parallel to equatorial plane in the Figure 1 through *A-B line*, placing it on the *ABC'D'* horizon, we get the position depicted in Figure 3. On that, the point *M'* is the folded position of *M*, in which the sundial stick intersects the equatorial plane on the horizon. Point *O* represents the point where the universe axis intersects the horizon, and *MEO* right triangle represents noon plane lied over the horizon. Since the *ME* edge of this triangle gives the direction of the universe axis, the *MEO* angle is equal to the complementary of the latitude of the present location: < (*MEO*) = (90° -  $\varphi$ ). If we take any < (*EMF*) = <(*EM'F'*) = h hour angle into consideration, the hour line that corresponds to this angle on the horizon is *OF*, and the hour angle is < (*EOF*) = *s*. Yet, as a corollary to the Figure 3, the following is found:

 $\begin{aligned} (\tan h) &= (EF)/(EM') \\ (EO) &= (EM)/(\sin \varphi) = (EM')/(\sin \varphi) \\ (\tan s) &= (EF)/(EO) = [(EF)/(EM')].(\sin \varphi) = (\sin \varphi).(\tan h) \\ (\tan s) &= (\sin \varphi).(\tan h) \end{aligned}$ 

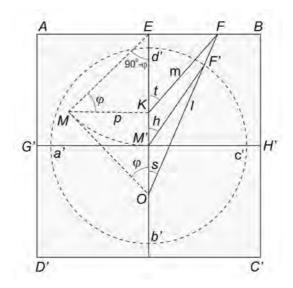


Figure 3. Angles of Hour lines and their lengths in horizontal sundial.

According to this, given the stick length p = MK, hour angles *s* and lengths l = OF can be calculated as following:

$$s = \tan^{-1} [(\sin \varphi).(\tan h)]$$
(1)  
and since is  $EK = p.(\tan \varphi), KO = p/(\tan \varphi)$   
$$l = OF = EO/(\cos s) = (EK + KO)/(\cos s)$$
  
$$= p.[(\tan \varphi) + 1/(\tan \varphi)]/(\cos s) = p/[(\cos s).(\sin \varphi).(\cos \varphi)]$$
  
$$l/p = 2/[(\cos s).(\sin 2\varphi)]$$
(2)

is found. However, in *ruhame*, all hour angles and lengths are calculated with reference to the point *K* where *MK* stick is perpendicularly located to the horizon. As a result,  $t = \langle EKF \rangle$  hour angle and m = KF length need to be expressed in terms of other hour parameters. If sinus theorem is applied to the *KOF* triangle, since [sin (180° - *t*)] = (sin *t*),

$$(\sin t)/l = (\sin s)/m$$

this expression, together with (2) above and  $(\cos t) = (EK)/m = (p/m).(\tan \varphi)$ 

 $(\tan t) = (\sin t)/(\cos t) = [(l/m) (\sin s)]/[(p/m).(\tan \varphi)] = [(l/p).(\sin s)]/(\tan \varphi)$ 

$$= (\sin s)/[(\cos s).(\sin \varphi).(\cos \varphi).(\tan \varphi)] = (\tan s)/(\sin \varphi)^2$$
  
$$t = \tan^{-1}[(\tan s)/(\sin \varphi)^2]$$
(3)

is found. Taking *EFK* right triangle into consideration, since *EF* = m.(sin t) and *EK* = p.(tan  $\varphi$ )

$$m^{2} = EF^{2} + EK^{2} = m^{2} .(\sin t)^{2} + p^{2} .(\tan \varphi)^{2}$$
  
or  
$$m/p = (\tan \varphi)/(\cos t)$$
(4)

can be written. Hence, since s and  $\varphi$  is known, t and (m/p) can be calculated.

However, these expressions are valid only for hour lines'  $F = I_i$  points on the equinox line. Throughout the year, sun rays reaches to the earth with an angle of declination of  $\delta \le \varepsilon = 23^{\circ}.5$ . Two times a year, on March 21 (the beginning of Aries) and on September 21 (the beginning of Libra), the slope angle is  $\delta = 0$ . In these dates, shade length is on the equinox line all day, and nights and days are equal. On June 21 (the beginning of Cancer) we have  $\delta = \varepsilon = 23^{\circ}.5$  and summer solstice is the longest day of the year. On the other hand, on December 21 (the beginning of Capricorn) we have  $\delta = \varepsilon = -23^{\circ}.5$  and winter solstice is the shortest day of the year. In all days except for equinoctial ones, the shade of the stick makes a hyperbolic curve on sundial's plane.

To draw the *ruhāma*, calculation of the shade points denoting  $I_i$  especially during June 21 and December 21 (beginning of Cancer and Capricorn, respectively) is necessary. For this, since we know that in the drawing projected onto the horizon plane, the shade falls on the *ME* line in equinox day, that line is lengthened, and since we know that in this situation the shade length is equal to *OF* line, we draw a circle with the center *O* and radius *OF*, and point *F'* is found (Figure 4). On the stick plane that is lied on the horizon, *OF'* line is equal to hour line with respect to its lengths. If we take <  $(F'MF_y) = \varepsilon$  and <  $(F'MF_o) = \varepsilon$  angles and intersect them with *OF'* line, *OF'*  $_y = l_y$  corresponds to shade length at the beginning of the Cancer, and  $OF_o' = l_o'$  corresponds to the shade length at the beginning of the Cancer. Since the real position of these shades is on the *OF* hour line, by using the arcs with center *O* and radius  $OF_y = l_y$  and  $OF_o = l_o$ , the real  $F_y$  and  $F_o$  points on *OF* can be obtained. Yet, angles and distances given in *Ruhāma* are  $t_y$  and  $t_o$  angles and,  $m_y$  and  $m_o$  distances defined with reference to *K* point.

To calculate these angles and distances, first the lengths of  $OF_y = l_y$  and  $OF_o = l_o$  should be calculated. Since in *MF'O* triangle, OF' = OF = l, when sinus theorem is applied, < (*MF'O*) = *w* angle can be calculated:

 $w = \sin^{-1} \{1/[(l/p) (\sin \varphi)]\}$ 

Similarly, by taking exterior angles into account, it can be seen that for  $MF_y'O$  triangle, < (MF'O)=  $w_y = (w + \varepsilon)$  and for  $MF_o'O$  triangle < ( $MF_o'O$ ) =  $w_o = (w - \varepsilon)$ . Accordingly, if sinus theorem is applied to  $MF_y'O$  and  $MF_o'O$  triangles, respectively

 $l_v = \{p.(\cos \varepsilon)\}/\{(\sin \varphi).[\sin (w + \varepsilon)]\}$ 

 $l_{o} = \{p.(\cos \varepsilon)\}/\{(\sin \varphi).[\sin (w - \varepsilon)]\}$ 

can be found. To calculate the K-centered angle and distances, first, sinus theorem should be calculated for < (*KFO*)= *x* in *OFK* triangle:

 $x = \sin^{-1} \{ [p.(\sin s)] / [m (\tan \varphi)] \}$ 

As a result, if the expression

 $(\sin t_y) = (l/p).[\sin (t_y + x - t)].(\tan \varphi)$  obtained from sinus theorem applied to  $OKF_y$  triangle is arranged,

 $t_{y} = \tan^{-1} \{ [\sin (x - t)] / \{ [(\cot \varphi) / (l_{y} / p)] - [\cos (x - t)] \} \}$ 

and from sinus theorem applied to  $KFF_v$  triangle

 $m_v/p = [(l_v/p).(\sin s)]/(\sin t_v)$ 

is calculated. Similarly when sinus theorem is applied to OKF triangle,

 $(\sin t_o) = (l_o/p).[\sin (x - t + t_o)].(\tan \varphi)$ 

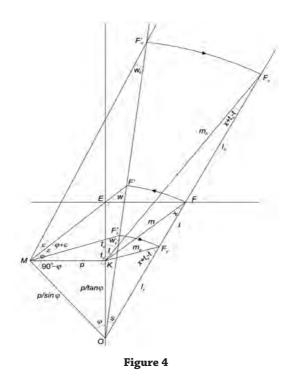
 $t_{o} = \tan^{-1}\{[\sin(x-t)]/\{[(\cot \varphi)/(l_{o}/p)] - [\cos(x-t)]\}\}$ 

can be found. And from the sinus theorem applied to KFF<sub>a</sub> triangle

 $m_{o}/p = [(l_{o}/p).(\sin s)]/(\sin t_{o})$ 

can be calculated.  $t_y$  angles and  $m_y$  shade lengths indicate  $Y_i$  Cancer points, and  $t_a$  angles and  $m_a$  shade lengths indicate  $O_i$  Capricorn points.

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All these calculated values are valid for zawali hours. If we want them to be valid also for  $ghur\bar{u}b\bar{i}$  hours that take sunrise or sunset as starting point as is the case for  $ruh\bar{a}ma$ , it should be taken into account that in Istanbul in the beginning of Cancer, compared to the equinox point where day and night is equal, sunrise and sunsets are early and late, respectively, as the amount of half-day remnant F. In contrast, in the beginning of Capricorn, compared to the equinox point, in Istanbul, sunrise and sunset are late and early, respectively, as the amount of half-day remnant F. (Figure 5)

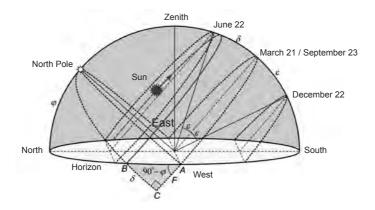


Figure 5

In solstices, considering  $AB = 90^{\circ} - \varphi = 49^{\circ}$  ve  $BC = \varepsilon = 23^{\circ}.5$  in Istanbul, half-day remnant F, is calculated as  $F = \sin^{-1} [(\tan \varphi).(\tan \varepsilon)] = 22^{\circ}.2$ . Accordingly, to calculate  $Y_i$  and  $O_i$  points, equation of daylight should be added to or subtracted from  $h_i$  angles. The expressions for calculation of these points are equal to the ones used in calculating  $I_i$  points.

#### 2.1.1. Drawing Hour Lines Elapsed after sunrise [Dā'ir min al- shurūq] and Hour Lines remaining to sunset [Dā'ir ilā al- ghurūb]

For this,  $Y_i$  points of the beginning of Cancer, and  $O_i$  points of the beginning of Capricorn should be calculated. Yet, to find these, first,  $I_i$  equinox points, then  $I_{yi}$  Cancer and  $I_{oi}$  Capricorn zawālī shade points, and then  $I_{yyi}$  Cancer and  $I_{ooi}$  Capricorn ghurubi shade points need to be calculated.

Using the expressions derived above, let us give which relevant relations are used to calculate the values that give the important points of a horizontal sundial that is to be used in Istanbul:

Since for equatorial sundials, each hour corresponds to  $15^{\circ}$  degrees, after the sunrise, angles regarding each hour on the equinox line being as i = 1,2,...6

$$h_i = 90^\circ - i.15^\circ$$

should be taken. As Istanbul's latitude is  $\varphi$  = 41°, from the expression (1), horizontal hour angles can be found from

 $s_i = \tan^{-1} [0,656.(\tan h_i)]$ 

and from the expression (2) shade lengths with reference to equinox points can be found from

 $l_i/p = (2.01966)/(\cos s_i)$ 

When these values are calculated with reference to *K* point where the stick stands, with the help of (3) and (4), for  $t_i$  angle and  $m_i/p$  shade lengths

$$t_i = \tan^{-1} [(2.32335).(\tan s_i)]$$
  
 $m_i/p = (0.86929)/(\cos t_i)$ 

is obtained. Since in the work of  $Ruh\bar{a}ma$ ,  $t_i$  angles are taken from the west point with reference to south it is evaluated as  $(90^\circ - t_i)$ . Table 1 shows  $t_i$  angles of  $I_i$  shade points on the equinox line and m/p distances.

Та	ble 1	1					
i	$I_{i}$	$\boldsymbol{h}_i$	<b>s</b> <sub>i</sub>	l <sub>i</sub> /p	$\boldsymbol{t}_i$	(90° - t <sub>i</sub> )	m <sub>i</sub> /p
1	$I_1$	75°	67°.7820	5 <sup>P</sup> .8090	80°.0289	9°.9711 <sup>≡</sup> 09° 58′ 16″	5 <sup>p</sup> .0204≡ 5 <sup>p</sup> 01' 44"
2	$I_2$	60°	48°.6488	3 <sup>p</sup> .0570	69°.2528	20°.7472 <sup>≡</sup> 20° 44' 50"	2 <sup>p</sup> .4539≡ 2 <sup>p</sup> 27' 14''
3	$I_3$	45°	33°.2649	2º.4154	56°.7304	33°.2696 <sup>≡</sup> 56° 43' 49"	1 <sup>°</sup> .5846≡ 1 <sup>°</sup> 35' 04''
4	$I_4$	30°	20°.7438	2 <sup>p</sup> .1597	41°.3462	48°.6548 <sup>≡</sup> 48° 39' 14"	1 <sup>p</sup> .1579≡ 1 <sup>p</sup> 09' 28"
5	$I_5$	15°	9°.9693	2 <sup>p</sup> .0506	22°.2144	67°.7956 <sup>≡</sup> 67° 47' 44''	0 <sup>p</sup> .9890≡ 0 <sup>p</sup> 59' 20''
6	$I_6$	0°	0°	2 <sup>p</sup> .0197	0°	90°	0 <sup>p</sup> .8693≡ 0 <sup>p</sup> 53' 57"

Since for  $I_{\gamma}, ..., I_{11}$  points, which are on the east side of the *ruhāma*, sundial drawing is symmetrical with reference to north-south line,  $t_i$  angles are taken with reference to west point.

To calculate  $Y_i$  and  $O_i$  points, which are time after sunrise and time to sunset, respectively, the equation of daylight with the degree of 22°.2 needs to be added to or subtracted from  $h_i$  angles. Other expressions can be used same as in the calculation of  $I_i$  points. Table 2 shows the comparison of calculated and given points.

Tab	le 2					
i	<b>Y</b> <sub>i</sub> / <b>O</b> <sub>i</sub>	<b>h</b> <sub>i</sub>	$(90^{\circ} - t_{yi})$	m <sub>yi</sub> /p	$(90^{\circ} - t_{oi})$	m <sub>oi</sub> /p
1	<b>Y</b> <sub>1</sub> / <b>O</b> <sub>1</sub>	97°.2/52°.8	- 22°/- 22° 46'	5 <sup>p</sup> .8090	80°.0289	09 <sup>p</sup> 58'16"
2	<b>Y</b> <sub>2</sub> / <b>O</b> <sub>2</sub>	82°.2/37°.8	– 13°/– 13° 32'	3 <sup>p</sup> .0570	69°.2528	20 <sup>p</sup> 44' 50"
3	<b>Y</b> <sub>3</sub> / <b>O</b> <sub>3</sub>	67°.2/22°.8	- 04°/- 05° 31'	2º.4154	56°.7304	56 <sup>p</sup> 43' 49"
4	<b>Y</b> <sub>4</sub> / <b>O</b> <sub>4</sub>	52°.2/07°.8	6°/4°	2 <sup>p</sup> .1597	41°.3462	48 <sup>p</sup> 39' 14"
5	<b>Y</b> <sub>5</sub> / <b>O</b> <sub>5</sub>	37°.2/0°	20°/15°	2 <sup>p</sup> .0506	22°.2144	$67^{P} 47' 44''$
6	<b>Y</b> <sub>6</sub> / <b>O</b> <sub>6</sub>	22°.2/0°	36°/ 25° 12'	2 <sup>p</sup> .0197	0°	90 <sup>p</sup>
7	<b>Y</b> <sub>7</sub> / <b>O</b> <sub>7</sub>	7°.2/0°	0°			
8	Y <sub>s</sub> /O <sub>s</sub>	0°	0°			
9	<b>Y</b> <sub>9</sub> / <b>O</b> <sub>9</sub>	0°	0°			
10	Y <sub>10</sub> /O <sub>10</sub>	0°	0°			

#### 2.1.2. Drawing the Tropics of Cancer and Libra

The angle and lengths of the shade falling upon the equinox line for Ariel and Libra can be obtained by the line connecting  $I_1 = I_{11}$  points. The calculated angle  $(90^{\circ} - t_1) = (90^{\circ} - t_{11}) = 09^{\circ} 58^{\circ}$  and shade length  $m_1 = m_{11} = 12.(5^{\circ} 01^{\circ} 44^{\circ}) = 60^{\circ} 20$  of these two points are equal to the one given in the text. (See *Table 1*)

#### 2.1.3 Drawing Hour lines to Midday and Hours after Midday [Fażl al- dāʻir]

To draw the hour lines for hours to and after midday, beginning points of  $I_{yi}$ . Sagittarius and  $I_{oi}$  Capricorn which corresponds to  $I_i$  equinox points that are formulated with reference to *K* points given above need to be calculated. From the expressions given above, these relations can be calculated as follows:

$$\begin{split} & w_i = \sin^{-1} \left[ (1,5243)/(l_i/p) \right] \\ & l_y/p = (1,3978)/[\sin (w + 23,5^\circ)] \\ & l_o/p = (1,3978)/[\sin (w - 23,5^\circ)] \\ & x = \sin^{-1} \left\{ \left[ (1,1504).(\sin s_i) \right]/(m/p) \right\} \\ & t_{yi} = \tan^{-1} \left\{ \left[ \sin (x_i - t_i) \right]/\{ \left[ 1/[(0,869)(l_{yi}/p)] \right] - \left[ \cos (x_i - t_i) \right] \} \right\} \\ & t_{oi} = \tan^{-1} \left\{ \left[ \sin (x_i - t_i) \right]/\{ \left[ 1/[(0,869)/(l_{oi}/p)] \right] - \left[ \cos (x_i - t_i) \right] \} \right\} \\ & m_{yi}/p = \left[ (l_{yi}/p).(\sin s_i) \right]/(\sin t_{yi}) \\ & m_{oi}/p = \left[ (l_{oi}/p).(\sin s_i) \right]/(\sin t_{oi}). \end{split}$$

The values regarding the shade angles (90° -  $t_{y(6-i)}$ ) and (90° -  $t_{o(6-i)}$ ) of  $I_{yi}$  and  $I_{oi}$  points that are evaluated, and the shade lengths of  $m_{y(6-i)}$  and  $m_{o(6-i)}$  are compared to those given in the text in *Table 3*:

Table 3						
Hour	Hours to Midday	Hours after Midday	$(90^{\circ} - t_{y(6-i)}) / (90^{\circ} - $	- t <sub>o(6-i)</sub> )	m <sub>y(6-i) /</sub> m <sub>o(6-i)</sub>	
			Calculated	Given	Calculated	Given
6	$I_{yo}$ - $I_{yo1}$	I <sub>y12</sub> -I <sub>y011</sub>	– 18° 10' / – 13°	- 18° 10' / - 13°	$44^{P}  17' /  62^{P}$	$44^{P}  17'  /  62^{P}$
5	I <sub>y1</sub> -I <sub>1</sub>	$I_{y11} - I_{11}$	- 08°.19' / 09° 58'	– <b>12</b> ° / -	24 <sup>p</sup> / 29 <sup>p</sup> 27 <sup>i</sup>	25° 15'/-
4	$I_{y2}^{-}(I_{2})-I_{o2}^{-}$	$I_{y10}$ - $(I_{10})$ - $I_{o10}$	- 00° 00' / 37° 09'	8° 22'/-	15°41'/	14 <sup>p</sup> /-
3	$I_{y3}$ -( $I_{3}$ )- $I_{o3}$	$I_{y9}^{-}(I_{9}) - I_{o9}^{-}$	10° 52' / 48° 15'	10° /	10 <sup>p</sup> 33'/	-
2	$I_{y4}^{-}(I_{4}^{-})-I_{o4}^{-}$	$I_{y8} - (I_8) - I_{o8}$	25° 65'/ 60° 51'	25° /	7º 05'/	6° / -
1	$I_{y5}^{-}(I_5) - I_{o5}^{-}$	$I_{y7}^{-}(I_{7})-I_{07}^{-}$	49° 44' / 74° 56'	49° 44'/	4 <sup>p</sup> 40'/	58° 45' / -
0	$I_{y6}^{-}(I_6) - I_{o6}^{-}$		90°	-	3° 47' /	-

#### 2.1.4 Drawing the Curves of Asr-i Awwal and Asr-i Thāni

In *ruhame*, there are two curves for *asr* prayer time defined as *asr-i awwal* and *asr-i thāni*. If the length of sundial stick *p* and the highest point of the Sun  $m_6 = (a-b)$  are given, the shade length for *asr-i awwal* in that day can be calculated by  $a_1 = (b-d) = (m_6 + p)$ , and that of *asr-i thāni* can be calculated by  $a_2 = (b-e) = (m_6 + 2.p)$  (Figure 6). If the distances are proportioned to the stick length *p*, as is the case in *ruhāma*, the following expressions can be obtained:

 $a_1/p = (m_6/p + 1)$  $a_2/p = (m_6/p + 2).$ 

Since the midday shades defined with reference to the point *K* are already calculated for equinox and tropics, it is easy to find  $a_1/p$  and  $a_2/p$  distances. Yet, to find the  $t_1$  and  $t_2$  angles, the expressions given above should be calculated in accordance to these distances.

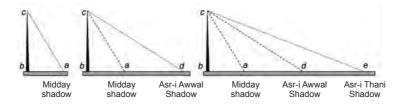


Figure 6 Definitions of the Time of Asr-i Awwal and Asr-i Thani

Table 4 shows the calculated and shade lengths given in the text, for the points of *asr-i awwal* and *asr-i thāni*. Some of the values given in the original text, which aremarked with red in the table are incorrect. They could be the result of incorrect copying (*istinsākh*).

For the  $A_i$  points on the equinox line, from the expression (4)

$$t_i = \cos^{-1}[(\tan \varphi)/(a_i/p)],$$

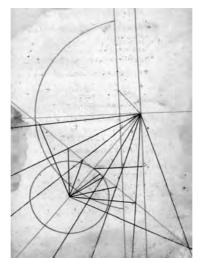
The easiest way to determine  $A_{iy}$  and  $A_{io}$  points that are at the beginning of Cancer and Capricorn is determining taiy and taiy angles on the drawing of *ruhame* by drawing circles with K as the center and aiy and aio as radiuses, and by guessing their intersection points with the hyperbolic curve. Otherwise, hyperbolic equations regarding Cancer and Capricorn need to be found and intersected with the aforesaid circle. *Table 5* summarizes the results regarding  $t_{aiy}$  and  $t_{aio}$  angles.

Table 4					
Point	Definition	Calculated		Given	
$A_{_i}$	a <sub>i</sub>	a <sub>i</sub> /p	<b>a</b> <sub>i</sub>	<b>a</b> <sub>i</sub>	
$A_{1y}$	$a_{1y}/p = m_{y6}/p + 1$	1 <sup>P</sup> .32	15 <sup>p</sup> 47'	15 <sup>p</sup> 47'	
$A_1$	$a_1/p = m_6/p + 1$	1 <sup>P</sup> .87	22 <sup>p</sup> 26 <sup>i</sup>	22 <sup>p</sup> 26'	
$A_{_{1o}}$	$a_{1o}/p = m_{o6}/p + 1$	3 <sup>p</sup> .10	37 <sup>p</sup> 35'	36 <sup>p</sup> 48'	
$A_{2y}$	$a_{2y}^{\prime}/p = m_{y6}^{\prime}/p + 2$	2 <sup>p</sup> .32	27 <sup>p</sup> 47'	26 <sup>p</sup> 48'	
$A_2$	$a_2/p = m_6/p + 2$	2 <sup>P</sup> .87	34 <sup>p</sup> 26'	34 <sup>p</sup> 26'	
$A_{2o}$	$a_{2o}^{\prime}/p = m_{o6}^{\prime}/p + 2$	4 <sup>P</sup> .10	49 <sup>p</sup> 10'	49 <sup>p</sup> 10'	

Table 5			
Point	Expression	Calcuated or Measured	Given
$A_{i}$	<b>t</b> <sub>ai</sub>	$90^{\circ} - t_{ai}$	$90^{\circ} - t_{ai}$
$\overline{A_{_{1y}}}$	-	0° 10'	0° 10'
$A_1$	$t_{a1} = \cos^{-1}[(\tan \varphi)/(a_1/p)]$	$27^{\circ}.70 = 27^{\circ} 42^{\circ}$	27° 48'
A <sub>10</sub>	-	57°	56° 48'
$A_{2y}$	-	-11°	-11° 32'
A <sub>2</sub>	$t_{a2} = \cos^{-1}[(\tan \varphi)/(a_2/p)]$	17°.63 ≡ 17° 38'	09° 26'
A_20	-	49°	49° 10'

#### Conclusion

Based upon the results obtained above, if a table of angles and distances of hour lines is prepared without any mistake, it is very easy to practically draw a horizontal sundial or *ruhāma* for Istanbul. Since in the drawing the author added to the end of the text, that the north-south line or midday line is drawn with a slope of 45 degrees to the right side of the paper, it gives a false impression to the reader that it is a vertical sundial (*Picture 3*). Even in vertical sundials east-west line is drawn parallel to the lower edge of paper. On the other hand, in horizontal sundials, equinox line is usually drawn parallel to the lower edge of paper. Given that, it is still a mystery how the author calculated these values and wrote the text.



**Picture 3** Sundial drawing at the end of the text.

11 In the original text, 10° is written instead of 10<sup>′</sup> by mistake.

Atilla Bir, Şinasi Acar, Mustafa Kaçar, A Mathematical Analysis of the Theory of Horizontal Sundials in the Ottoman Period: The Case of Risālah of Ruhāma

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### Appendix

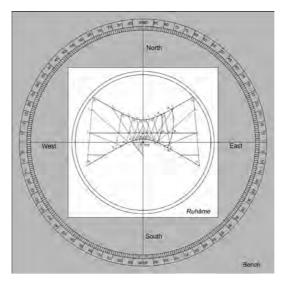


Figure A: Board used in drawing ruhāma

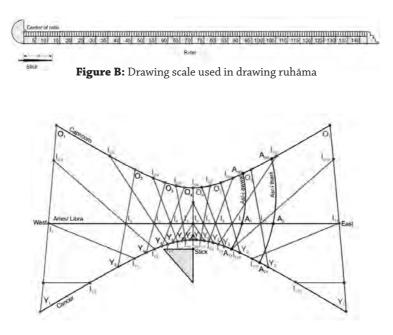
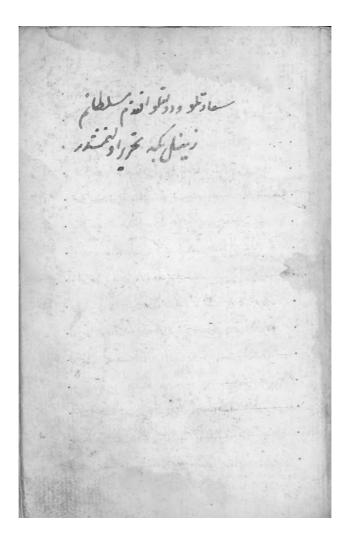


Figure C: Points determined in drawing sundial on ruhāma

## Facsimile of Ruhāma



NAZARİYAT Journal for the History of Islamic Philosophy and Sciences

جراست الداول دآيو التربع ايدوب جاتادهم ور المالية ومن الم نك اساميسن ياذه سن دورت بع اولور بري دج شما الس المالي التراكي ليهشرتي وبري دبع شمالي عنهي وبري دبع جنوبيه الحميد نته رتب المسالماني والصَّلوة والسَّادم علي محمَّة والذ شرقي وبري دبع جنوبي غربي اولوري وهم بربهي وصيداجعين لكك برفقه وحق بورساله وقسطنطيه طقسان درجه يه تقسيم ايدوب هر يتر درجه نك عضنه رخامه وضواتمانا يجون تجربرا يترم فيرآمجن التدوارقام بازوسن الملهج شمالي شرقي بالعدا مهاله لوكه بادست لراكة تاخطا اوزمع بولدم تجربه دنك ابتداسني نقطه وسترقدن نقطه وشماله بغاية اولنماسش بونقي بجه دفعته تجربه وبرهان الدوكدنصكو بولد ورج شمالي عزبي نا مبدا ونقطه ومعربد ندس بورساله يازدمكه عطاليه يتشه اسان وجه نقطه وشمالده نهايت بولور ومربع جنوبي شرقي نك اوزرم جامدوضع ايده ولوحقري دخى دعاء عيردن نقطه يستهتمن وربج جنوبي وغزبي فلنقطة مزيز اونتم وف الابردوزرنده المش تحنه نا نقطه دجنوسين نهات بولوراس دورت مح الاستر. اون تهيزن بخامه معداري اوتوب جقروب رخامة اوزم وضع اولندق بنصكن مشامد التخادير آنانايجنه عكر بركيدوب مخامنان سطى اول بردايؤه وصنع اولنه الذن ايجر وبودايوه دانخ ومغ اولن تختنك سطحنه موآذي اوله اندن صكرة برجامنك برى يونيد بعرى يستول اختيا برك و داولد بودائره لوى طسترة كح يختد اوستنده اوكان دا يرم مركز ندن اول تخته نا اوستناق مركادله مر دايز ا

اور تهسنده برمقاس کد یک سق اول طولی بر مسطو نلااون ایکی درجه سی مقداریا ولور و اول مقا ملت ظلی می طوعه قسط طینه عرضنده یو دقیق اوچ درجه اولود به امه دن طنی دویش اما طلاعد برساعت کوب ایکنی سات اول ده بیتن بش درجه ویکری بردیقت اولی کرک بر تاکه مل ایت اما کا بادان ویکری بردیقت ول اسلی دیل دان ایجون مخ اول ان اوست دور دمید قسی بر امدی مسط ولات اما کا بادان اور دای بر دیمه محکم شویله معلوم اول که مرضامه مرکن بر ایک دا یا له بر کره سن اول در محم وردی دو بر جرف ها چاول در که سط وزین محم مرضامه مرکن بر ایک دا یا له بر کره سن اول در محم ورد دو بر بر جرف ها چاول در که سط وزین محم ورد دو بر بر جرف ها چاول در که سط وزین محم ورد دو بر دو جرف من خوای محمل وزین محمل ورد دو بر دو جرف من دویت اجرا و در محمل وزین محمل ورد دو بر دو جرف من دویت اولی در محمل و در دور محمل و در محمل ورد دو بر دو جرف در محمل و در محم

تقسيمندن قسمتايدهسن بش باخوداونردوجيه بخه اختيا وايد رسانا يجنه اسامىء بلهآن بازه سن نخرافن كوره وبأخود بودآبزه لري مدآرآت تمام وضع اولنتغذ مكرة وضع ايدةسن هرقتي سن اختيارا يدوسك بالزدرف المسطع وضواتمك بالندهد طريقى ولدركه نحاسدن بابرغدن وبالخود جشابة يوفقه جق رجد ولكبى نسته ايده سنكثادن يوز قرق اوج درجيه تقشيم ايدهس بتردرجيه ومايخود اوتردرجيه اعداد مادهسن ابتكاسنه بردلوك دله ست كماول دلول مركزه سطع اولور بوغلان كمتو صلا هان بومتالدر بوبلجه يوزق تاوج درجيه وارغبه تقسيم ايدوب اعداد بادهسن وبوزقرق اوجد تمايت بولدوغنك سبجى ودركم شاد بخامدنك

Atilla Bir, Şinasi Acar, Mustafa Kaçar, A Mathematical Analysis of the Theory of Horizontal Sundials in the Ottoman Period: The Case of Risalah of Ruhame

> قو به سن كداول فقطه آول جديدن برساعت تحوي المحفى ساعتك ولحاولور انذن سيطان اقلنده وصعاولنان نقطه ايله بوحديجا ولنده وضع اولنان نقطه اوستنه جدول قيوب برخطمستقير جكهن كدسقيا سالباشنك ظلى بوخطة كلبته جع مدل تده طلوعدن أكين اعتك اولخاولود الذن صكرة دون دو به جنوبى عرب اوسته اخراء دائره دونقطه يسغ بدن ۱۰ درجه و۲۳ دقيقناوستنهقوب واخراءمسطرون اس ددجه دقيق حازيسته بزنقطه قويه سن كدمد آرسطانده اوجنجساعتك وكحلولور اندنهسطة ودونده روب مهج شمالي وغزيو اوستنه اجراء وايؤه دن نقطة مغربه ٥ دوجه و وقيق اوزرنيد قوي واخواد مسطع دن - درجه و١٠ دقيقة محازدينه بونقطه قويه سن كه مدار جديدن اوجنج ساعتلا ولجاولوب ومكريط نده وصحاولنان نقطه إيله بوملا وجديد وصع اولنان is

نقطه اوستنه جدول توب بونطاستقيم لخ بجريسك بحير مراقل اوبني سلمتا الولي اور او دن مسطوء دورنده وب مهم جنوني غزيج اوستنه نقطه دمز بدنا بزاء در برده وب مهم جنوني غزيج اوستنه نقطه دمز بدنا بزاء مسطوه دو بنه درجه و ۱۰ دقيقا وزمينه قوب واجزاء مسطوه دو بنه درجه و ۱۰ دقيقا وزمينيه قوب المعنام مسطوه دو بنه دب مرجع شمالي مغزيو او سنته بن نقطه مغربين اجزاء دايره دم به درجه ۲۰ دقيقه او زمينه قوب معلم جدين دو درجه ساعتان ولي ولور سر المخاطه اوستنه جدول قوب بخط مستقيم بكه سكه يحيى معلم ادة و در بح ساعتان ولي ولور سر المخاطه اوستنه جدول قوب بخط مستقيم بكه سكه بحيى معل ادة و در بح ساعتان ولي اولور المان مسطوه بنه دوجه ما در بي اوستنه ابزاء والي و ده نقطه م مغربين م دوجه او زمينه قوب والمود المان مسطوه مع درجه مع اد نوسته بر نقطه مق يه سنكه معلم مع اد درجه مع اد نوسته بر نقطه مق يه سنكه مع مراري مها اده

بشيخ ساعتك ولحاولور اندن مسطر ودوند رب بعشا لى غزى اوزىرند اجراء دائر د فقطه مغربان دوجه و ۲۰ دقيقاوستنه قوي واجراء سطره دن . درجه ۲۹ دقیقه محاردسته برنقطه قربه سنک ممآد جديدن بشبخ سلمتك أولج اولور اندن ينه بواتجى نقطه اوستنه جدول قويب برخطمستقيم جكه سنكه جيعى مدلراتده بتبخ ساعتك اولياولور انذنمسطر ودرنده رب بج شمالي عزبي اوستنه اجراء دايره ده نقطه مغريده ادرجه اوستنه قوب واخرآ دمسطره دن ب درجه ٢٠ دققه ما زيينه برنقطه قو به سنكه مدآد سهائد التخ ساعتك ولجاداور اندنه سطرو وينه دونده رب شمالجا بنه اولان خطَّان عقالتها وك رآت اوستنهق بب واجراء مسطودنه ، درجه ماز سب خطانصفالتهاداوستنده برعلامت ايدوسن اندنجدول قويب بوعلامت اندوكان يردن اول فقطيه برخطامستقيم 6.

بحکسن که معن مداراتده البخی ساعتلا اولی اولور ا بذن مسطوع دوند وب مربع خمالی عزیمی اوستنه اجرآد دایژه ده نقطه و مغربون مربع خمالی عزیمی اور اوز مربیه قوب واجرا و مسطود ن ، درجه مر ، دقیقه مازیسنه برفعله قویه سنکه مدارسطانده بد بخی ساعتل اولی ولور اندن مسطوع دوند رب ماست نف القه ارلیا وسنه قوب واجرآ و سطود ن درمیه اندن اول نقطه ایله بوعاد مارست ، مرعلامت ایری برخط بکه سنکه معاد اوست ، مرعلامت ایری اولور اندن مسطوع دونده دب بنه دمیع خمالی عزیمی اولور اندن مسطوع دونده دب بنه دمیع خمالی عزیمی موستند ، اجرآ در ایزه ، در غذه مربع خمالی عزیمی موستند ، اجرآ در ایزه ، دفقطه و مغرب ، درجه م مارست ، موقوب واجرا و مسطوع دن ، درجه م مارست ، بر مقطه قویه سنکه مدار مربان دسکریمی مارست ، بر مقطه و به سنکه مدار مربان در میه م مارست ، بر مقطه و به سنکه مدار مربان ده سکریمی ساعتلاق وله راد اندن مسطوع دور ، مرب م م دور می م مارست ، بر مقطه و به سنکه مدار مربان ده مرب مربه م مارست ، بر مقطه و به سنکه مدار مربان ، درجه ،

 ۱۰۰ درجه وا ، د فقه حازیت ، برنقطه قو به سن
۱۰ در بونقطه دن اول نقطیه جدول قرب برخقاستیم چکه سنکه جمع مدارات ه استبول عرض د مقیاست براسنان تلکي بوخله ستیمه کلهان عرب برساعت قالور ایزن مسطم و یه درج جنوبی د شرقی اوستنده اجرآ و دایو ده نقطه ه مشرقدن ۱۰ درجه و ۲۰ دقیقه مازیسته برنقطه قو به سن ای ن سطح و دو نقیه م می تعلقه ایله اول می ای دن سطح و دو نقه د بر مع تمالی شرقی اوستاده ، جرآه دایو ده نقطه ه مشرق بر مع تمالی شرقی اوستاده ، جرآ و ده نقطه ه مشرق د بر مع تمالی مقرق اوستاده ، جرآه دایو ، ده نقطه مشرق د بر مع تمالی مقرق اوستاده ، جرآه دایو ، ده نقطه مشرق د بر مع تمالی مقبله مدول قویب بر مقله مشرق است مست که نظله تباس بو مقله کلسه عزو به ایکی ساعت اجراه دایو ده نقطه دامشرقدن ، درجه و ۲۰ دقیقا اوز به اجراه دایو ده نقطه دسترقدن ، درجه و ۲۰ دقیقا اوز به اور داند ، اجراه دایو ده نقطه دسترقدن ، درجه و ۲۰ دقیقا اوز به اور داند ، اجراه دایو ده نقطه دامشرقدن ، درجه و ۲۰ دقیقا اوز به اور داند ، اجراه دایو ده نقطه دامشرقدن ، درجه و ۲۰ دقیقا اوز به اور به اور به اور داند ، درجه او مع دقیقا اور به اور منامی اور به می داند ، درجه او اجراه دایو ده نقطه دامشرقدن ، درجه و ۲۰ دقیقا اور در به اور به ایکی ساعت اجراه دایو در مع می درجه و ۲۰ دقیق اور داند ، درجه و ۲۰ دقیق اور داند ، درجه و ۲۰ دقیق اور دار داند ، درجه و ۲۰ دقت اور داند ، درجه و ۲۰ دقیق اور داند ، درجه و ۲۰ دقیق اور داند ، درجه و ۲۰ دقیق اور داند ، درجه و ۲۰ دقیق اور داند ، درجه و ۲۰ دقیق اور داند ، درجه و ۲۰ دقیق اور داند ، درجه و ۲۰ دقیق اور داند ، درجه و ۲۰ دقیق اور داند ، درجه و ۲۰ دقیق اور داند ، درجه و ۲۰ دقیق اور داند ، درجه و ۲۰ دقیق اور داند ، درجه و ۲۰ دقیق اور داند ، درجه و ۲۰ دقیق اور درجه اور داند ، درجه و ۲۰ دقیق اور داند ، درجه و ۲۰ دقیق اور داند ، درجه و ۲۰ دقیق اور داند ، درجه و ۲۰ دقیق اور در داند ، درجه و ۲۰ دقیق اور در درجه اور در درجه اور در درجه اور در درجه اور در درجه اور در داند ، درجه و ۲۰ د درجه و ۲۰ د دقیق اور در درجه و ۲۰ درجه و ۲۰ د درجه و ۲۰ د دور دند ، دور در داند ، در درجه و ۲۰ دقیق اور در در درجه و ۲۰ در درجه و ۲۰ در در درجه و ۲۰ در درجه و ۲۰ در در درجه و ۲۰ در در در در داند ، درمو دام در در درمی می مرد در

> قويب واجرايمسطردن ١١ درجه و٩ دقيقه صاريسينه برنقطة قوب الذنه سطروء دونده دب سنه درج شمالئ شرقجاوستنده اجرآء رايره ده نقطه وسترقدن الدرجه و٠٠ دقيقه اوزبه بنه قويب وابزآ دمسطى دن ٠٠ درجه محاذبيت برنقطه قويه سن الذن اول نقطه ايله بونقطيه جدول قوب برخط سنقيم جكه سنكه ظل مقياس بخط كلدكم غروبه اوج ساعت قالور اندن مسطره دويده مه بنه م جنوبى وشرقي اوستد واجراء دا بوه د . نفطه وسترقدن ودرجه ودقيفه اوزم بنه قويب ماجراء سسطع دندا درجه محاذبيت برنقطه قويسن الدن مسطرة دونده دب ربع شمالية شرقي اوستده الزاء دايوه ده نقطه ومشرقدن ٨٠ دوجه و٨ ٠٠ دقيق ٢ اوستنه قويب وأجزا ومسطره دن ۲۰ درجه و ۲۰ دقيقه مادسته برنقطه قويه سن اندن اول نقطه ابله يوتقليه جدول قوب برخط ستقتم جكه ستكه ظلامقداس بوخطله the state

كليجك غروبه ديون ساعت قالور اندنه سطع ودونده در ديع تمالي د شرقي اوستده اجزا تودا يوه ده فطله شقون يه ۱۰ درجه اوستنده قوب اجزا تودا يوه ده فطله شقون دقيق حاذيت مرفط مقريه سن اندن ستا مف التها ل مناغ جابندن بشيم ساعت يجز اندر كل خطلا نصب مناغ جابندن بشيم ساعت يجز اندر كل خطلا نصب بمه سن كه ظل ميان بونده بونطيه مدول قوب برخط مربع شمالي شرقي اوستده اجزا ودايزه ده فقط شتر يدن م بع شمالي شرقي اوستده اجزا ودايزه ده فقط شتر يدن م بع شمالي شرقي اوستده اجزا ودايزه ده فقط شتر يدن م بع شمالي شرقي اوستده اجزا ودايزه ده فقط شتر يدن م بع شمالي شرقي اوستده اجزا ودايزه ده فقط شتر يدن م دوجه و اد دقيقه ها دوسنده برفقت قو يه سن اندن م استداد اول نقطه او سنه ما دول قوب بر مناج يسكه م لي ترام بو مقل كلي من يوب التي ما مت قالورا تدن م سطحه دودندوب ديج شمالي ه شرقي اوستاده فقطه و Atilla Bir, Şinasi Acar, Mustafa Kaçar, A Mathematical Analysis of the Theory of Horizontal Sundials in the Ottoman Period: The Case of Risalah of Ruhame

الحاد الميز آنجز باله طريق ولدركم مسطوع خطافت التها وليصاغ جابندن ديع نما لغ عزيد اوستده ١ جزاء ولي ده نقطه مخبد ١ درجه و ٥ وقت اوزينه قوب مستقيم اوستده برعلامت ايده سن ارذن مسطع وزيد مستقيم اوستده برعلامت ايده سن ارذن مسطع وزيد دي خطف التها ولتصول جابندي مع نمايي متولي شر وي وابؤا يسطعون ، درجه و ٠ وقت ما ذونبي نم وتب وابؤا يسطعون ، درجه و ٠ وقت ما ذونب اول خطامت عم اوستده ، برعلامت ايده سن بوابي ي مكتم كر كواول خذه ، درجه و ٠ وقت ما ذورب مكتم كر كواول خذه ، درجه و ما وقت ما ذورب مكتم كر كواول خذه ، درجه و ما وقت ما ذورل مكتم كر كواول خذه ، درجه و ما ولي مان مكتم كر كواول خذه ، درجه و ما وتنه ما ورب مر خطامت عرب اور المحال الميزان دورل من الدا تر او مله ما والما والما واستده ، درما ولي مغريا ما الدرل شو يا كم مقاطه لي مالجال وستده من ما ولي مغريا من موال الميزان در لر شويله معلوما ولرف المحكوم خدين الولي مغريا الما يرا شويله معلوم الد

مشرقدن اجزاءدابو دن ۱۷ درجه وقعتماوز منه قوب واجرآءمسطرودن م درجه محادست برنقطه قوبرست اندن خطاضف المنهادل صاغ جابنتدن متقبل ولان خطك باشدد بونقطيه مدول قوب برخط جكمهن كه ظامعياس بوخطه كلحك غروبه بدهيساعت قالوريس مخاسه نك صول جابفة آخايره ابله تمام اولدي سيد لكك حدميع خطوطك بالتلري بربريده متصل ولمتى كرك اكرجنوب واكرتمال جابنحه وجدول قويب بربرينه ستقل اتملت. جائزدر أماحشى يوبه اولما زحسفا يوجه اولسون دىرسانجنوب حاجا تحون براكر بعه حدول ايده سنكه جمي خطوطك باشلونيه برآبرجه مرورايدن براكريجه خطبكه سنكماكامل سطان ديل إنذن يرجدول دالنى بيلا ايدوسنكر شمالها ندند واولان خطوطك آب بالشلرنيه مرورايده اندن ينهبراكريه خطح كمستكه اولخطهمد ومرصى ديرلس الم يسايد ذصك ومدار

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طريقيني باذايده لوطريقي اولدركه مسطره دربع جنوبى و غربي اوستنده اجراء دايرة نفقلهم فيدندا درجه وا دقيقراوزمرينه تؤبب واجزآ ومسطره دنجم درجه و٧١ دقيق مازست مدآرسهات اوزمزده برعلامت ايده سناندن مسطود دونده رب وبعجزي وغزي اوزرن فقطة مغربة ١٢ درجهاوستنه قويب واجزاء مسطود ن٦٠ درجه محاذسينه طلوعدن الجني ساعتك ولنده حكدوكك ساء خطاستقيم اوزمرنده برعلامت ليدوب اندن لواتكى علامتك اوستنه جدول قويب سرطه برقزل خطجكه سنكه مقياسك باشنك ظلى وقزل خطه كلجك زواللاتى ساعت قلور اندن مسطود دوندور ربح جنوبي غزيي اوستنده اخراءدا يرمه ده نقطه مغردن درجه اوستنه قوب واجرآء مسطودن كه به حازيستهمد ببطان وبرعادت ايدهسن اندن بوعات اللهمال الحل بالتح متصل ولدوغى مره جدول قوبوب برقزل خطبكي سنكه فلل مقياس وتول فقله كليمان نصفا لمهاره 3.

بنى ساعت قاور شىدىد نى كرة بو فى ايزى ومن اتكم ا سا ن طريق اولدوك مدام الحمل الملايان ابله دايز خطاري كه تعالملح انتر در فى مدام الحمل الملايان ابله دايز خطاري علامت اندكد نعب كرة جد ولك بولوجن اول علامت اوستنده و براوجن تعالمل اون يوه قويب بو قزل خطاري جكمس بس اند ت سطوه دو ندوب اج جنوبي عزيي اوستنده انقطه ه مزيد مر معان بسنه مدام جرائد برعالامت ايدوب ندن بنه متعالم إيله بوعلامت اوستنه جدول قويب برقزل خط بنه متعالم إيله بوعلامت اوستنه جدول قويب برقزل خط الزايه مسطوه من خطامت قريم باور اله دورت ساعت الغرائد ن مسطوه منا مالم يمان دين مرعالامت ايدوب ندن مالوراندن مسطوه منطام تلجيل دوالله دورت ساعت الغرائي مسطود من دوب ممان بي ماده الرجساعة قالوراندن سنكه فل مقياس بكما له عزيها او جساعت قالوراندن مسطوه دورد درب وم متما له عزيها وستده انقطه محرية

الددرجه اوزيرينه قوب واجراء سطرودن ودرجه مع ويت المارسطانة برعلامت ايدو بداولكي برقول خطجكه سنكه ظليقاس كاكليك وواله ايكى ساعت قالور اندنمسط ودوندرب ينه ربع شمالي وغزى ده نقطه ومغريدت سطر درجه دقيقماوستده قرب واجراء. مطردن محمه مازمينهما بهطانده برعادمت ايدوب ينهبر قزلخط جكه سنيكه ظليقياس بوخط كلحك دوالهبرساعت قالوربس خطاصت التهارلنصاع جابنى تمام اولدي اتما شويله معلوم إواسونكم بوقزل خطله قبالآزوال زواله قلاف بادرر ومعدالي وال زوالدن كجهني بلدرزنته كرساه خطلمة الآروال كحمنى وبعد الزوالغروبه فادنى بالدر فص الربيل ندن صكر مسطرة خطانصف النهاران صولجا تبندن ربع شماليء شرقي اوستند اجرآ ودائره ده نقطه بسشرقدن سط مد درجه ودقيقتا وستنه قويب واجزا ومسطره دن مع

كەظلىقياس توكاكلىكەن زوالدن دورت تشاعت تجمش اولور الدنهسط ، اجزاء دائر ودن اوسنه قرب واحرا ومسطع دن كه به ما زست مدارسطانده بر عدوسايدوب اندن بوعلامت ايله ممل عمل والمعيزان باشنه مدول قريب برخط جكه سنكه ظابقياس بوكاللحك زوالدن نتر ساعت عيش إولوراندن سطر اجراءدا يؤهدن نقطه مشرقين مردجه اوستنه قويب اخرا مسطرون سد درجه محازمينه غروب برساعت قلان خطهستقيم وستنده برعلامتا بداس اندن مسطع ودونده وباجزاء دائره دن نقطه ومشرقدن 3- اوستنه قوب واجرا وسطره دف د رما زيستهمدآ وسطانده وعلامت ابدوب اندن بوايكى علامتاوستنه جدول قريب برقز لخط جكه سنكه ظلمقاس بوخطه كلعك زوالدنالتى اعت تجنواواور يسرمخامه نات دايرى وفضل الدايرى متام اولدعاندن .50

صكرة مآبر وفضل الدائي خطلونيك اوستنه وقبلون بازمسن فس محيكك مهلم ايك قور وضح ايدة سن برنيه عصالول وبريد عصائلي دولومقياسك فللي قوس عصالول محيك المامين قدده عصالياك وقتى اولور وقوس عصالية محيك المامين قدده عصالياك وقتى المحيد قوس عصاليا محيك المامين ماده معالية قدن عصر المحيد قوس عصالية لمحيك المدين ما عظم المدي قوس عصالية لمحيك المدين ما عظم مربع جنوبي وشرقي اوستنده اجزاء دائي ود فقطه مشقية محيد ويقد محمل والمحين والمول من ما معام ووقيق ما زيست ممال محيان الده برعالا مالية والمراد معطوه دون درب حيالي فشرقيا وستنده اجزاء دائي ود معطوه دون درب حيان مين معال محاور بنيه قوت المراد دائي و معطوه دون كر حمان دوست معال المحاول الموالية المراد برعالات الدوست الدن مسطوة دون درب مع شالي شرقي اوستده اجزاء داخت المحاد قون دائي اوست قوب Atilla Bir, Şinasi Acar, Mustafa Kaçar, A Mathematical Analysis of the Theory of Horizontal Sundials in the Ottoman Period: The Case of Risalah of Ruhame

& الكلكه بالمه ناندكونه مقيال وضع واجزآ وسطرودن او محاز دسته مرارب اوستنده بر علامتايده سناندن بركامى فتحايدو بواوج علامته مرد التمليك للطريق اولدركه نحاسدن باخود برغدن ايدو · اوج الحدمان وسيورجه اوله رخامه نا حركزنيه · ايدىنچە رخامە اوستىدە مركزىن ارانوب برقوس ايدەست · وضرايده سن الماطولى رخامه تل سطىندن » اوستنه عصاوله يوبازه سن وعصرتان مطربق اولدركه · بقاروسط زنا ونابكى بجدمة دارى · مسطرودوندن وبع جنوبي شرقيا وستنه اجراء دايرده · اولەزىادە اولىيە ونقصان اولىيەيس. نقطة مشرقدت الددرجه ودقيقداوستنه قوب واجزآء · وصخابتكريخ المه اوستنداو لان · مسطوده كوو محازمين المرار المارة موعلامتا يدهسن · دائرة نان دورجانندن بركادله اندن مسطرة دوندرب ربع شمالي فشرقيا وستنه اجرآه . مقياسانياش اولي بين برآبر دآيرهده نفطةمتز قدن مركوا وستنه قويب واجزاء · اولدقد تما يتوقي الله مسطع دن لدكوها زيسته مما رالحل والمبزآن اوستنه برعلامتا بددسن اندن مسطرع دوندوب دبع تمالى . ، وضعاولمشاولور ، . شوطيطورولم شرقيا وستنها جرآد دائوه ده نقطة سترقدن مط اوسنه قوب واجزآ ومسطره دن مط حاز سنه مدلم جدى اوستنده برعلامتايده سناندن عصاقلده كحكبى بر قوس دآخايد اس اوستنه عصراني ديو بازهس is

